Assignment 0
Due on 2020-09-14, 23:59 IST.

1. Explain the concept of vector space \((V, +, \cdot)\) over a field \(F\), where \(V\) is equipped with operations \(+: V \times V \to V\) called vector addition and \(\cdot: V \times F \to V\) called scalar multiplication. Recall the axioms that \(V\) needs to satisfy in order to be called a vector space. Write the following is a vector space? \(V = \{\mathbf{v} \in \mathbb{R}^2 \mid \mathbf{v} \cdot \mathbf{v} = 0\}\).

2. A linear system of \(m\) equations in \(n\) unknowns can be modeled as \(Ax = b\), where \(A\) is an \(m \times n\) matrix, \(x\) is the \(n\)-vector column of unknowns, and \(b\) is a \(m\)-vector column vector. The goal is to determine a \(\tilde{x}\) which satisfies the following:
(a) \( \tilde{x} = \mathbf{0}\), then the system has a unique solution.
(b) \( \tilde{x} = \mathbf{0}\), then the system has a unique solution.
(c) \( \tilde{x} = \mathbf{0}\), then the system has a unique solution.

3. Diagonalize any symmetric and positive definite matrix. Consider the symmetric matrix \(A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\). Find the eigenvalues \(\lambda\) and eigenvectors \(\mathbf{v}\) of this matrix.

4. This course deals with algorithms for a lot of number theoretic problems. As a computer programmer it is clear that you must have written a program to test whether a number is a prime or not. You would like to know that in this course you will learn very fast algorithms for primality testing (e.g. Rabin primality testing for one such beautiful algorithm). Before that remove your money with the following warm-up question.

What is the following code: 

```python
def is_prime(n):
    if n <= 1:
        return False
    if n <= 3:
        return True
    if n % 2 == 0 or n % 3 == 0:
        return False
    i = 5
    while i * i <= n:
        if n % i == 0 or n % (i + 2) == 0:
            return False
        i += 6
    return True

is_prime(29)  # Returns True
```

What is the following list give minimum possible value of \(x\), for the above program to work correctly?

- \(x = 2\)
- \(x = 3\)
- \(x = 4\)
- \(x = 5\)
- \(x = 6\)

Answer: \(x = 5\)