Unit 14 - Week 12

Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-09, 23:59 IST.

Let $M_1 = \begin{bmatrix} 2 & -1 & 3\sqrt{2} \\ 0 & 1 & \sqrt{2} \\ 0 & 0 & 3 \end{bmatrix}$, $M_2 = \begin{bmatrix} 2 & 1 & 3\sqrt{2} \\ 0 & 1 & -\sqrt{2} \\ 0 & 0 & 3 \end{bmatrix}$, $M_3 = \begin{bmatrix} 2 & 0 & 3\sqrt{2} \\ 1 & 1 & \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$, $M_4 = \begin{bmatrix} 2 & 0 & 3\sqrt{2} \\ -1 & 1 & -\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$, $M_5 = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ and $M_6 = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

Recall that two matrices A and B are said to be **unitarily equivalent** if there exists a unitary matrix U such that $A = UBU^*$. Now, use the above matrices in Problems 1 and 2 to conclude that the upper triangular matrix obtained in the "Schur's Lemma" need not be unique.

1) Which among the following is an INCORRECT Option? *1 point*

- M_1 and M_2 are unitarily equivalent
- M_3 and M_6 are unitarily equivalent
- M_5 and M_6 are unitarily equivalent
- M_2 and M_5 are unitarily equivalent

No, the answer is incorrect.
Score: 0
Accepted Answers:
M_5 and M_6 are unitarily equivalent

2) Which among the following is an INCORRECT Option? *1 point*

- M_3 and M_4 are unitarily equivalent
- M_1 and M_5 are unitarily equivalent
- M_4 and M_6 are unitarily equivalent
- M_2 and M_3 are unitarily equivalent

No, the answer is incorrect.
Score: 0
Accepted Answers:
M_2 and M_3 are unitarily equivalent

3) Let $M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & -1 & \sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Then, which among the following is TRUE? *1 point*

- M and N are unitarily equivalent
- M and N are NOT unitarily equivalent
- M is diagonalizable but N is NOT diagonalizable
- M is NOT diagonalizable but N is diagonalizable

No, the answer is incorrect.
Score: 0
Accepted Answers:
M and N are unitarily equivalent

4) Let $M = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 10 & 9 \\ -4 & -2 \end{bmatrix}$. *1 point*
Then, which among the following is an INCORRECT Option?

- M and N are similar
- There exists a unitary matrix U such that $M = UNU^*$
- The eigenvalues of M and N are the same.
- M and N are NOT diagonalizable

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exists a unitary matrix U such that $M = UNU^*$

5) Which among the following is an INCORRECT Option? *1 point*

- If M is Hermitian then $\mathbf{rank}(M) =$ the number of non-zero eigenvalues of M
- If M is skew-Hermitian then $\mathbf{rank}(M) =$ the number of non-zero eigenvalues of M
- There exists M such that $\mathbf{rank}(M) \neq$ the number of non-zero eigenvalues of M
- If $M^2 = M$ then $\mathbf{rank}(M) \neq$ the number of non-zero eigenvalues of M

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $M^2 = M$ then $\mathbf{rank}(M) \neq$ the number of non-zero eigenvalues of M

6) Let M be a normal matrix with real entries. Then, which among the following is an INCORRECT Option? *1 point*

- If all eigenvalues of M are 0 then $M = \mathbf{0}$
- If all eigenvalues of M are 1 then $M = I$
- If $M = (m_{ij})$ has $\sigma(M) = \{\lambda_1, \ldots, \lambda_n\}$ then $\sum_{i,j} |m_{ij}|^2 = \sum_i |\lambda_i|^2$
- M has n real orthonormal eigenvectors

No, the answer is incorrect.
Score: 0
Accepted Answers:
M has n real orthonormal eigenvectors

7) Let M be a normal matrix with $(\lambda, \mathbf{x})$ as an eigen-pair with $\lambda \neq 0$. Then, which among the following is an INCORRECT Option? *1 point*

- $(\bar{\lambda}, \mathbf{x})$ is an eigen-pair for M^*
- $M^*\mathbf{x}$ is an eigenvector of M corresponding to λ,
- $M\mathbf{x}$ is an eigenvector of M corresponding to λ
- If M is unitary then $|\lambda| = 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$M\mathbf{x}$ is an eigenvector of M corresponding to λ

8) Let $M \in \mathbb{M}_n(\mathbb{C})$ be an Hermitian matrix. Then, which among the following is an INCORRECT Option? *1 point*

- x^*Mx is real for each $x \in \mathbb{C}^n$.
- M has real eigenvalues.
- S^*MS is Hermitian for each $S \in \mathbb{M}_n(\mathbb{C})$.
- The eigenvectors of M corresponding to distinct eigenvectors need NOT be orthogonal

No, the answer is incorrect.
Score: 0
Accepted Answers:
The eigenvectors of M corresponding to distinct eigenvectors need NOT be orthogonal

9) Let $M \in \mathbb{M}_n(\mathbb{C})$. Then, which among the following is an INCORRECT Option? *1 point*

- If $n = 5$ and $M^{15} = \mathbf{0}$ then $M^5 = \mathbf{0}$
- If $\mathbf{x}^*M\mathbf{x} = 0$, for all $\mathbf{x} \in \mathbb{C}^n$ then M need NOT be $\mathbf{0}$
- If $\mathbf{x}^*M\mathbf{x} = 0$, for all $\mathbf{x} \in \mathbb{C}^n$ then $M = \mathbf{0}$
- Let $\{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$ be an orthonormal set in $\mathbb{R}^n$. Then $M = \sum_{i=1}^{k} \mathbf{u}_i\mathbf{u}_i^T$ is diagonalizable

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $\mathbf{x}^*M\mathbf{x} = 0$, for all $\mathbf{x} \in \mathbb{C}^n$ then M need NOT be $\mathbf{0}$

10) Let $M = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. If $10M^{-1} = \alpha M^2 + \beta M + \gamma I_3$ then the value of $\gamma - \beta$ equals ... *1 point*

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 24.9,25.1