Unit 13 - Week 11

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-02, 23:59 IST.

1) Let $M \in \mathbb{M}_n(\mathbb{R})$ be an invertible matrix and let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ with $\mathbf{x} \neq \mathbf{0}$ and $\mathbf{y}^T M^{-1}\mathbf{x} \neq 0$. Define $N = \mathbf{x}\mathbf{y}^T M^{-1}$. Then, which among the following is an INCORRECT Option? *1 point*

- $\lambda_0 = \mathbf{y}^T M^{-1}\mathbf{x}$ is an eigenvalue of N of multiplicity 1
- $1 + \alpha\lambda_0$ is an eigenvalue of $I + \alpha N$ of multiplicity 1, for any $\alpha \in \mathbb{R}$.
- 1 is an eigenvalue of $I + \alpha N$ of geometric multiplicity $n-2$, for any $\alpha \in \mathbb{R}$.
- $\det(M + \alpha\mathbf{x}\mathbf{y}^T)$ equals $(1+\alpha\lambda_0)\det(M)$, for any $\alpha \in \mathbb{R}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
1 is an eigenvalue of $I + \alpha N$ of geometric multiplicity $n-2$, for any $\alpha \in \mathbb{R}$.

2) Let $M, N \in \mathbb{M}_2(\mathbb{R})$ such that $\det(M) = \det(N)$ and $\mathrm{trace}(M) = \mathrm{trace}(N)$. Then, which among the following is an INCORRECT Option? *1 point*

- M and N have the same set of eigenvalues.
- M and N need NOT be similar.
- M and N are similar.
- No statement can be made about similarity of M and N

No, the answer is incorrect.
Score: 0

Accepted Answers:
M and N are similar.

3) Let $M = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \$and\$ N = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. *1 point*

Then

- both M and N are diagonalizable
- M is diagonalizable but N is NOT diagonalizable
- neither M nor N are diagonalizable
- M is NOT diagonalizable but N is diagonalizable

No, the answer is incorrect.
Score: 0

Accepted Answers:
M is NOT diagonalizable but N is diagonalizable

4) Let $M \in \mathbb{M}_n(\mathbb{C})$. Then, which among the following is an INCORRECT Option? *1 point*

- If $M + \alpha I_n$ is diagonalizable for some $\alpha \in \mathbb{C}$ then M is diagonalizable
- If M is diagonalizable then $M + \alpha I_n$ is diagonalizable for all $\alpha \in \mathbb{C}$
- If $\mathrm{alg.mul}_t(M) = m$ and $\mathrm{Rank}(M - tI_n) = n - m$, for all $t \in \sigma(M)$ then M is NOT diagonalizable.
- If $\mathrm{alg.mul}_t(M) = m$ and $\mathrm{Rank}(M - tI_n) = n - m$, for all $t \in \sigma(M)$ then M is diagonalizable.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $\mathrm{alg.mul}_t(M) = m$ and $\mathrm{Rank}(M - tI_n) = n - m$, for all $t \in \sigma(M)$ then M is NOT diagonalizable.

5) Let $M, N \in \mathbb{M}_n(\mathbb{C})$. Then, which among the following is an INCORRECT Option? *1 point*

- If M is similar to N then M is diagonalizable if and only if N is diagonalizable
- If M and N are diagonalizable using the same matrix S then they are similar
- If M and N are diagonalizable and if $\sigma(M) = \sigma(N)$ then M is similar to N
- If M is invertible then NM and MN are similar

No, the answer is incorrect.
Score: 0

Accepted Answers:
If M and N are diagonalizable using the same matrix S then they are similar

6) Let J be an $n \times n$ matrix with all entries 1. Then, which among the following is an INCORRECT Option? *1 point*

- $\mathrm{Geo.Mul}_n(J) = \mathrm{Alg.Mul}_n(J) = 1$
- $\mathrm{Geo.Mul}_0(J) = \mathrm{Alg.Mul}_0(J) = n - 1$
- If $M = (a-b)I_n + bJ$ then $a - b$ is an eigenvalue of M
- If $M = (a-b)I_n + bJ$ then $a + nb$ is an eigenvalue of M

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $M = (a-b)I_n + bJ$ then $a + nb$ is an eigenvalue of M

7) Let $T : \mathbb{R}^5 \longrightarrow \mathbb{R}^5$ be a linear operator with $\mathrm{Rank}(T - I) = 3$ and $\mathrm{Null}(T) = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 \mid x_1 + x_4 + x_5 = 0,\ x_2 + x_3 = 0\}$. Then, which among the following is an INCORRECT Option? *1 point*

- The eigenvalues of T are 0 and 1
- $\mathrm{Geo.Mul}_1(T) = 2$
- $\mathrm{Geo.Mul}_0(T) = 2$
- T is diagonalizable

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathrm{Geo.Mul}_0(T) = 2$

8) Which among the following is an INCORRECT Option? *1 point*

- If $M^2 = M$ then M is diagonalizable
- If $M^2 = \mathbf{0}$ and $M \neq \mathbf{0}$ then M is NOT diagonalizable
- $\begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$ is diagonalizable
- $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ is diagonalizable

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ *is diagonalizable*

9) Let $M \in \mathbb{M}_n(\mathbb{R})$ with $\mathrm{Rank}(M) = 1$. Then, which among the following is an INCORRECT Option? *1 point*

- $M = \mathbf{x}\mathbf{y}^T$, for some non-zero vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.
- M has at most one non-zero eigenvalue of algebraic multiplicity 1
- M is diagonalizable whenever $\mathrm{trace}(M) = 0$
- M is NOT diagonalizable whenever $\mathbf{y}^T\mathbf{x} = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
M is diagonalizable whenever $\mathrm{trace}(M) = 0$

10) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ such that $\{\mathbf{u}, \mathbf{v}\}$ is a linearly independent set. Define $M = \mathbf{u}\mathbf{v}^T + \mathbf{v}\mathbf{u}^T$. Then, which among the following is an INCORRECT Option? *1 point*

- M is a symmetric matrix
- $\dim(\mathrm{Null}(M)) = n - 2$
- $\mathrm{Geo.Mul}_0(M) = n - 2$
- $\mathrm{Geo.Mul}_0(M) \geq n - 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathrm{Geo.Mul}_0(M) \geq n - 1$