1. Consider the simple linear regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \ i = 1, 2, \ldots, n, \ \epsilon_i \sim N(0, \sigma^2) \) are identically and independently distributed. The parameters \( \beta_0, \beta_1 \) and \( \sigma^2 \) are estimated by ordinary least squares estimation where 

\[
\hat{\beta}_1 = \frac{s_{xy}}{s_{xx}}, \ \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \ s_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}), \\
s_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2, \ s_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2, \ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \ \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \text{ and } \hat{y}_i \text{ is the } i^{th} \text{ fitted value.}
\]

The variance of \( \hat{\beta}_0 \) is specified by

(i) \( \frac{1}{n-2} \left( \frac{1}{n} + \frac{s_{xx}^2}{s_{xy}} \right) \sum_{i=1}^{n}(y_i - \hat{y}_i)^2. \)

(ii) \( \frac{1}{n-2} \left( \frac{1}{n} + \frac{s_{xx}^2}{s_{xy}} \right) \sum_{i=1}^{n}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2. \)

(iii) \( \frac{1}{n-2} \left( \frac{1}{n} + \frac{s_{xx}^2}{s_{xy}} \right) (s_{yy} - \hat{\beta}_1^2 s_{xx}). \)

(iv) \( \frac{1}{n-2} \left( \frac{1}{n} + \frac{s_{xx}^2}{s_{xy}} \right) (s_{yy} - \hat{\beta}_1 s_{xy}). \)

A. (i) and (ii).
B. (ii) and (iv).
C. (ii), (iii) and (iv).
D. (i), (ii), (iii) and (iv).

2. If the random errors \( \epsilon_i \)'s having zero mean and unknown variance \( \sigma^2 \) in the simple linear regression model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \ i = 1, 2, \ldots, 40 \) do not follow the normal distribution, then which of the following test is used to test the significance of null hypothesis \( H_0 : \beta_1 = 5? \)

A. \( t \) test.
B. \( Z \) test.
C. Any one of the \( t \) test or \( Z \) test.
D. None of the \( t \) test or \( Z \) test.
[3] Let \( \hat{s}_{xx}^* = \sum_{i=1}^{n} x_i^2 \), \( \hat{s}_{yy}^* = \sum_{i=1}^{n} y_i^2 \) and \( \hat{s}_{xy}^* = \sum_{i=1}^{n} x_i y_i \). The 100(1 - \( \alpha \))% confidence interval of \( \beta \) in the simple linear regression model \( y_i = \beta x_i + \epsilon_i \) where \( \epsilon_i 's \) are identically and independently distributed following normal distribution with zero mean and unknown variance \( \sigma^2 \) is

A. \( \left( \frac{s_{xy}^*}{s_{xx}^*} - \frac{t_{n-1}/2, \alpha/2}{\sqrt{(n-1)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} , \frac{s_{xy}^*}{s_{xx}^*} + \frac{t_{n-1}/2, \alpha/2}{\sqrt{(n-1)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} \right) \).

B. \( \left( \frac{s_{xy}^*}{s_{xx}^*} - \frac{t_{n-2, \alpha/2}}{\sqrt{(n-2)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} , \frac{s_{xy}^*}{s_{xx}^*} + \frac{t_{n-2, \alpha/2}}{\sqrt{(n-2)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} \right) \).

C. \( \left( \frac{s_{xy}^*}{s_{xx}^*} - \frac{t_{n-3, \alpha/2}}{\sqrt{(n-3)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} , \frac{s_{xy}^*}{s_{xx}^*} + \frac{t_{n-3, \alpha/2}}{\sqrt{(n-3)s_{xx}^*}} \sqrt{s_{yy}^* - \left( \frac{s_{xy}^*}{s_{xx}^*} \right)^2} \right) \).

D. None of these.

[4] The 95% confidence interval of \( \hat{\beta}_1 \) in the model \( y_i = \beta_0 + \beta_1 x_i + \epsilon_i \), \( i = 1, 2, \ldots, 7 \), \( \epsilon_i 's \sim N(0, \sigma^2) \) where \( \sigma^2 \) is unknown, is obtained for a given set of data as (10, 20). The tabulated value of \( t \) statistic at 5% level of significance with 5 degrees of freedom is 2.0. Which of the following statements are correct.

Statement 1 : \( \hat{\beta}_1 = 15.0 \) and standard error of \( \hat{\beta}_1 \) is 2.5.

Statement 2 : \( \hat{\beta}_1 = 15.0 \) and variance of \( \hat{\beta}_1 \) is 6.25.

Statement 3 : The null hypothesis \( H_0 : \beta_1 = 5 \) is accepted at 5% level of significance.

Statement 4 : The null hypothesis \( H_0 : \beta_1 = 5 \) is rejected at 5% level of significance.

A. Statements 1, 2 and 3 are correct.

B. Statements 1, 2 and 4 are correct.

C. Statements 1 and 3 are correct.

D. All the statements 1 and 4 are correct.
[5] A simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, 2, \ldots, n$ is fitted on the basis of given set of data. The fitted model is obtained as $y = 2 + 3x$. The interpretation of the value 3 is

A. when $x$ changes by one unit, then the average value of $y$ changes by 3 units.
B. when $x$ changes by one unit, then the value of $y$ changes by 3 units.
C. when $x$ changes by one unit, then the average value of $y$ lies in the interval $(0,3)$.
D. when $x$ changes by one unit, then the value of $y$ lies in the interval $(0,3)$.

[6] A simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, 2, \ldots, n$ is fitted on the basis of given set of data. The fitted model is obtained as $y = 6 + 7x$. The interpretation of the value 6 is

A. when $x = 0$, then the average value of $y$ is 6 units.
B. when $x = 0$, then the value of $y$ is 6 units.
C. when $x$ changes by 7 units, then the change in the average value of $y$ is 6 units.
D. when $x$ changes by 7 units, then the change in the value of $y$ is 6 units.

**Question 7 - 10 are based on the following output of a software which is obtained while fitting a simple linear regression model $y = \beta_0 + \beta_1 x + \epsilon$, $\epsilon \sim N(0, \sigma^2)$ to a given set of data.**

[7] The fitted regression model is

A. $y = 44.2 + 2.89x$
B. $y = 2627.8 - 37.15x$
C. $y = -37.15 + 2627.8x$
D. $y = 59.47 - 12.86x$
Figure 1: Software output for Questions 7 - 10

[8] The null hypothesis \( H_0 : \beta_1 = 0 \) at 5% level of significance is

A. accepted.
B. rejected.
C. nothing can be said.
D. Inadequate data.

[9] The null hypothesis \( H_0 : \beta_0 = 0 \) at 5% level of significance is

A. accepted.
B. rejected.
C. nothing can be said.
D. Inadequate data.
The least squares estimate of \( \sigma^2 \) is

A. 44.2
B. 2.89
C. 166255
D. 9236
Solution to Assignment 2

Answer of Question 1 – D

Answer of Question 2 – D

Answer of Question 3 – B

Answer of Question 4 – B

Answer of Question 5 – A

Answer of Question 6 – A

Answer of Question 7 – B

Answer of Question 8 – B

Answer of Question 9 – A

Answer of Question 10 – D