Curves and Surfaces: Solutions to Assignment-3

Qn 1

Translation does not change σ_u or σ_v — (1)

For a rotation about origin $(U\sigma)_u = U\sigma_u$

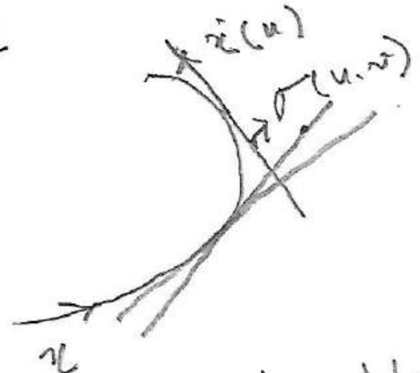
$(U\sigma)_v = U\sigma_v$ — (2)

and $U\sigma_u \cdot U\sigma_u = \sigma_u \cdot \sigma_u$

similarly for $\sigma_u \cdot \sigma_v$ and $\sigma_v \cdot \sigma_v$ — (2)

Qn 2

a)



$$\sigma(u,v) = \gamma(u) + v\,\dot{\gamma}(u)$$ — (3)

Tangent developables

b) $\sigma_u \times \sigma_v = (\dot{\gamma} + v\ddot{\gamma}) \times \dot{\gamma} = v\,\ddot{\gamma} \times \dot{\gamma}$, $\|\dot{\gamma}\| = 1$

Thus σ to be regular it is necessary that $\ddot{\gamma} \neq 0$

i.e. $\kappa = \|\ddot{\gamma}\| > 0$ at all points — (2)

now $\ddot{\gamma} = \kappa\eta$, η - principal normal

$\Rightarrow \sigma_u \times \sigma_v = -\kappa v \vec{b}$, $\vec{b}$ - binormal

Thus σ to be regular we must have $\kappa > 0$

and $v \neq 0$, i.e. γ is <u>NOT</u> part of the surface. — (2)

(c) $E = \|\sigma_u\|^2 = (\dot{\gamma} + v\ddot{\gamma}) \cdot (\dot{\gamma} + v\ddot{\gamma})$

$= \dot{\gamma}\cdot\dot{\gamma} + 2v\,\dot{\gamma}\cdot\ddot{\gamma} + v^2\,\ddot{\gamma}\cdot\ddot{\gamma}$

$= 1 + v^2 k^2$ — (1)

$F = \sigma_u \sigma_v = (\dot{\gamma} + v\ddot{\gamma})\cdot\dot{\gamma} = \underbrace{\dot{\gamma}\dot{\gamma}}_{=1} + v\,\dot{\gamma}\ddot{\gamma}$

$G = \sigma_v \sigma_v = \dot{\gamma}\cdot\dot{\gamma} = 1$ — (1)

FFF : $(1+v^2k^2)\,du^2 + 2\,du\,dv + dv^2$ — (1)

(d) There is a planar curve (unit speed) $\tilde{\gamma}$ with signed curvature k (done in Lecture). FFF of tangent developables to $\tilde{\gamma}$ is also given by $(1+v^2k^2)\,du^2 + 2\,du\,dv + dv^2$ — (3)

Since $\tilde{\gamma}$ is planar, tangent lines lies on planes. — (2)

Qn 3. The isometry is

$\sigma(u,v) \longmapsto \left(u\sqrt{2}\cos\frac{v}{\sqrt{2}},\ u\sqrt{2}\sin\frac{v}{\sqrt{2}},\ 0\right)$
$= \tilde{\sigma}(u,v)$ — (3)

verify this by calculating FFF's. — (2)

Qn 4

$$\mathrm{FFF}(\sigma) = (1 + 2v\,\dot{\gamma}\cdot\dot{\delta} + v^2\,\dot{\delta}\cdot\dot{\delta})\,du^2 + 2\dot{\gamma}\cdot\delta\,du\,dv + dv^2 \quad -(1)$$

So σ is conformal iff $1 + 2v\,\dot{\gamma}\cdot\dot{\delta} + v^2\,\dot{\delta}\dot{\delta} = 1$

and $\dot{\gamma}\cdot\delta = 0 \quad \forall u, v \qquad -(1)$

Thus $\dot{\delta} = 0 \implies \delta$ is constant

and $\dot{\gamma}\cdot\delta = 0 \implies \gamma\cdot\delta$ is constant $\qquad -(2)$

Say $= c$.

Thus σ is conformal iff δ is constant and γ is contained in the plane $-(1)$.

$\vec{r}\cdot\delta = c$

(Thus σ is a generalized cylinder.)