1. Express $x^2 + xy + y^2 = \text{in powers of } (x - 1) \text{ and } (y - 2). \text{ (Use Taylor’s formula)}$.

2. Let $f$ be real-valued and assume that the directional derivatives $D_u f(x + tu)$ exits for each $t \in [0, 1]$. Prove that for some $\theta \in (0, 1)$ we have $f(x + u) - f(x) = D_u f(x + \theta u)$.

3. If $f$ is real-valued and if the directional derivatives $D_u f(x) = 0$ for every $x$ in an open ball $B(x, \delta)$ and every directions $u$, prove that $f$ is constant on $B(x, \delta)$.

4. Investigate the following functions for maxima and minima or saddle points.
   (a) $f : \mathbb{R}^3 \rightarrow \mathbb{R}, f(x, y, z) = x^2 y + y^2 z + z^2 - 2x$.
   (b) $f : \mathbb{R}^3 \rightarrow \mathbb{R}, f(x, y, z) = (ax^2 + by^2 + cz^2)e^{-x^2-y^2-z^2}$.

5. Let $f : U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$, $U$ open, has continuous first and second order partial derivatives and $X \in U$ is a stationary point for $f$. Let $H = \left( \frac{\partial^2 f}{\partial x_i \partial x_j} \right)$ be the second derivative of $f$ at $X$. Denote by $H_k$ the $k$th principal minor of $H$.

   Prove the following:
   (a) If $\det H_k < 0$ for some $k = 1, 2, \ldots, n$ then $X$ is a saddle point.
   (b) If $\det H \neq 0$ then
      (i) $f$ has a local minimum at $X$ if and only if $\det H_k > 0$ for all $k$.
      (ii) $f$ has a local minimum at $X$ if and only if $(-1)^k \det H_k > 0$ for all $k$.
      (iii) $f$ has a saddle point at $X$ if and only if it is neither local maximum or minimum.
   (c) If $\det H = 0$ then the test is inconclusive (give an example).

6. Find the shortest distance from the point $(a, 0)$ to the parabola $y^2 + 4x = 0$. 
7. Find the point on the line of intersection of the two planes $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ which is nearest to the origin.