Assignment 10

The due date for submitting this assignment has passed.
As per our records, you have not submitted this assignment.

1) Let \( X_t \) for \( t \leq s \), denote the greatest integer less than or equal to \( s \).
Then, which of the following functions defined on \([0, m]\) can represent a sample path of a simple process?  
\[
\begin{align*}
S_1(s) &= s \\
S_2(s) &= s^2 \\
S_3(s) &= \lfloor s \rfloor \\
S_4(s) &= \lfloor 2s \rfloor
\end{align*}
\]

No, the answer is incorrect. Score: 0

Accepted Answers:
- \( S_3(s) \)

2) If the stochastic process \( \{X_t\} \) satisfies \( dX_t = -aX_tdt + \sigma dW_t, \) \( X_0 = 0 \), where \( a, \sigma \) are positive constants, and \( \{W_t\} \) is a Brownian motion, then which of the following is/are true?  
\[
\begin{align*}
X_t &= A(t) - \sigma \int_0^t \exp(-\sigma s) dW_s \\
X_t &= A(t) - \sigma \int_0^t \exp(-\sigma s) dt \\
X_t &= A(t) - \frac{\sigma}{\sigma^2 + 1} \\
X_t &= A(t) - \sigma \int_0^t \exp(-\sigma s) dW_s
\end{align*}
\]

No, the answer is incorrect. Score: 0

Accepted Answers:
- \( X_t = A(t) - \sigma \int_0^t \exp(-\sigma s) dW_s \)

3) State whether the following statement is TRUE or FALSE:  
If \( f = \int_0^t \sigma dW_s \) then the quadratic variation of \( f \) equals the variance of \( f \).  

TRUE

4) State whether the following statement is TRUE or FALSE:  
The quadratic variation of an \( \mathcal{F}_t \)-process is always equal to the quadratic variation of the corresponding \( \mathcal{F}_t \)-integrable process.  

TRUE

5) Let \( X_t = \int_0^t X_s dW_s \), where \( X_t \) takes the value 1 for \( 0 \leq t < 2 \), takes the value 3 for \( 2 \leq t \leq 3 \) and takes the value 0 otherwise.
Then, which of the following is/are true?  

\( \{X_t\} \) is a martingale.

No, the answer is incorrect. Score: 0

Accepted Answers:
- \( \{X_t\} \) is a martingale.

6) The process \( \{W_t^2 - t\} \) is a martingale.  

No, the answer is incorrect. Score: 0

Accepted Answers:
- \( \{W_t^2 - t\} \) is a martingale.

7) State whether the following statement is TRUE or FALSE:  
For \( X_t = W_t \) and \( Y_t = e^{\alpha t} \), we have that \( \mathbb{E}(W_t^2) = e^{\alpha^2 t} \).  

TRUE

8) State whether the following statement is TRUE or FALSE:  
For \( X_t = W_t \) and \( Y_t = e^{\alpha t} \), we have that \( \mathbb{E}(W_t^2) = e^{\alpha^2 t} \).  

No, the answer is incorrect. Score: 0

Accepted Answers:
- \( \mathbb{E}(W_t^2) = e^{\alpha^2 t} \)