

# Unit 10 - Week 9

## Course outline

How does an NPTEL online course work?

Week 1

Week 2

Week 3

Week 4

Week 5

Week 6

Week 7

Week 8

Week 9

Advanced Probability Theory (Lec22)

Advanced Probability Theory (Lec23)

Quiz : Assignment 9

Week 9 Feedback Form

Week 10

Week 11

Week 12

Download Videos

Assignment Solution

## Assignment 9

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-04-01, 23:59 IST.

$X_{(k)}$  represents the  $k^{\text{th}}$  order statistic throughout this tutorial sheet

1) Let  $X_1, X_2, \dots, X_{10}$  be 10 independent samples taken from  $U(0, 1)$  distribution. What is the expectation of  $X_{(2)}$ ?

1 point

- 1/10  
 2/10  
 1/11  
 2/11

No, the answer is incorrect. Score: 0

Accepted Answers: 2/11

2) Let  $X_1, X_2, \dots, X_{10}$  be 10 independent samples taken from  $U(0, 1)$  distribution. What is the variance of  $X_{(1)}$ ?

1 point

- 0.008  
 0.015  
 0.0069  
 0.0909

No, the answer is incorrect. Score: 0

Accepted Answers: 0.0069

3) Let  $X_1, X_2, \dots, X_{10}$  be 10 independent samples taken from  $Exp(1)$  distribution. What is the expectation of  $X_{(1)}$ ?

1 point

- 10  
 0.1  
 5  
 1

No, the answer is incorrect. Score: 0

Accepted Answers: 0.1

4) Let  $X_1, X_2, \dots, X_n$  be  $n$  independent samples taken from  $Exp(1)$  distribution. What is the expectation of  $X_{(r+1)} - X_{(r)}$  for any valid  $r$ ?

0 points

- $r+1$   
  $r$   
  $n-r$   
  $n-r-1$

No, the answer is incorrect. Score: 0

Accepted Answers:  $n-r$

5) Let  $X_1, X_2, \dots, X_n$  be  $n$  independent samples taken from  $Exp(\lambda)$  distribution. Which of the following statements are True?

1 point

- $X_{(1)}$  and  $X_{(2)}$  are independent  
  $X_{(r)}$  and  $X_{(r+1)} - X_{(r)}$  are independent  
  $X_{(3)} - X_{(1)}$  and  $X_{(5)} - X_{(4)}$  are independent  
  $X_{(r+1)} - X_{(r)}$  is distributed as  $Exp(\lambda(n-r))$

No, the answer is incorrect. Score: 0

Accepted Answers:

$X_{(r)}$  and  $X_{(r+1)} - X_{(r)}$  are independent  
 $X_{(3)} - X_{(1)}$  and  $X_{(5)} - X_{(4)}$  are independent  
 $X_{(r+1)} - X_{(r)}$  is distributed as  $Exp(\lambda(n-r))$

6) Let  $X_1, X_2, \dots, X_n$  be  $n$  independent samples taken from  $U(0, 1)$  distribution, then find the distribution followed by the  $k^{\text{th}}$  order statistic  $X_{(k)}$ ?

1 point

- Beta2( $k, n-k+1$ )  
 Beta1( $k, n-k+1$ )  
 Beta2( $n-k+1, k$ )  
 Beta1( $n-k+1, k$ )

No, the answer is incorrect. Score: 0

Accepted Answers: Beta1( $k, n-k+1$ )

7) Let  $X_1, X_2, X_3$  be a random sample from a continuous distribution having the pdf  $f(x)=2x, 0 < x < 1$  and zero elsewhere. Compute the probability that the smallest of  $X_1, X_2, X_3$  exceeds the median of the distribution.

1 point

- 1/8  
 1/4  
 1/2  
 1/6

No, the answer is incorrect. Score: 0

Accepted Answers: 1/8

Please follow the below paragraph to answer the following set of questions (Question 8, 9 and 10)

Let  $Y_1 \leq Y_2 \leq Y_3 \leq Y_4$  be the order statistics of a random sample of size  $n = 4$  from a distribution with pdf  $f(x) = 2x, 0 < x < 1$  and zero elsewhere. Answer the following questions.

8) Find the joint pdf of  $Y_3$  and  $Y_4$

1 point

- $16y_3^4 y_4$  for  $0 < y_3 \leq y_4 < 1$   
  $48y_3^5 y_4$  for  $0 < y_3 \leq y_4 < 1$   
  $64y_3^2 y_4$  for  $0 < y_3 \leq y_4 < 1$   
  $8y_3^3 y_4$  for  $0 < y_3 \leq y_4 < 1$

No, the answer is incorrect. Score: 0

Accepted Answers:  $48y_3^5 y_4$  for  $0 < y_3 \leq y_4 < 1$

9) Find the conditional pdf of  $Y_3$ , given  $Y_4 = y_4$

1 point

- $\frac{3y_3^4}{y_4^4}$  for  $0 < y_3 \leq y_4$   
  $\frac{4y_3^5}{y_4^4}$  for  $0 < y_3 \leq y_4$   
  $\frac{6y_3^2}{y_4^4}$  for  $0 < y_3 \leq y_4$   
  $\frac{7y_3^6}{y_4^4}$  for  $0 < y_3 \leq y_4$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\frac{6y_3^2}{y_4^4}$  for  $0 < y_3 \leq y_4$

10) Evaluate  $E[Y_3 | Y_4 = y_4]$

1 point

- $\frac{7y_4}{8}$   
  $\frac{5y_4}{6}$   
  $\frac{3y_4}{4}$   
  $\frac{6y_4}{7}$

No, the answer is incorrect. Score: 0

Accepted Answers:  $\frac{6y_4}{7}$