Assignment 12
Due on 2020-04-26, 23:59 IST.

1. There are 100 people at a plane. Let X be the weight (in kg) of the (i+1)th person on the plane. Suppose that the X_i are i.i.d., and E(X_i) = 70 and Var(X_i) = 8. Find the probability that the total weight of the 100 people on the plane exceeds 6500 kg.

2. Let X = \( \sum_{i=1}^{20} X_i \), where \( X_i \) are independent and identically distributed random variables with mean 1 and variance 0.1. What is the probability that \( \sum_{i=1}^{20} X_i \) exceeds 20?

3. The length of time, in hours, taken by a group of people to solve one math problem is normally distributed with mean 47 hours and standard deviation 8. The group received a memo to increase their speed. What is the probability that the group can solve the problem in less than 40 hours?

4. Prove that the sum of two independent normal random variables is also normal.

5. Let \( X_1, X_2, \ldots, X_n \) be i.i.d. random variables with mean \( \mu \) and variance \( \sigma^2 \). Suppose that \( \sum_{i=1}^{n} X_i \) exceeds \( n\mu + 3\sigma \). What is the probability that at least one of the \( X_i \) exceeds \( \mu + 3\sigma \)?

6. Suppose that you have a sample of 50 values from a population with mean \( \mu = 50 \) and standard deviation \( \sigma = 10 \). What is the probability that the sample mean is at least 55? What is the probability that the sample mean is at least 60? What is the probability that the sample mean is at least 56?

7. Suppose the age of cars on the road is normally distributed with mean 12 years. If the standard deviation is known to be 3 years, what is the probability that a randomly selected car has been on the road for more than 15 years?

8. The amount of sugar contained in a packet of sugar is normally distributed with mean 500 grams and standard deviation 10 grams. If a sample of 100 packets is taken, what is the probability that the mean weight of the 100 packets will be within 5 grams of the expected value?

9. The time required for a repair call is exponentially distributed with mean 2 hours. What is the probability that a repair call will require more than 3 hours?

10. A hospital has a queue system where patients arrive at a rate of 5 per hour. The service time is exponentially distributed with a mean of 20 minutes. What is the probability that a patient will have to wait for more than 10 minutes?

11. The number of customers arriving at a store follows a Poisson distribution with a mean of 10 per hour. What is the probability that exactly 5 customers will arrive in the next hour?

12. The number of calls received by a call center follows a Poisson distribution with a mean of 20 calls per hour. What is the probability that more than 30 calls will be received in the next hour?