Assignment 9

Due on 2019-02-22, 23:59 EST.

The due date for submitting the assignment has passed. As per our regulations, you have not submitted this assignment.

NOTE: In this assignment, answers that differ may be acceptable.

1. Let set \( A = \{x \in \mathbb{R}^2 | x_1 + x_2 = 0\} \) be a subset of a Hilbert space \( H \). Which of the following is true?
   - A. \( A \) is linearly independent.
   - B. \( A \) is linearly dependent.

2. Show that \( \mathbb{R}^2 \) is an inner product space with the following inner product:
   \[ \langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 + x_2y_2 \]

3. Let \( \{v_1, v_2, v_3\} \) be an orthonormal basis for a Hilbert space \( H \). Define set \( S = \{v_1, v_2, v_3\} \).
   - a. Show that \( S \) is linearly independent.
   - b. Show that \( S \) is linearly dependent.
   - c. Show that \( S \) is a basis.
   - d. Show that \( S \) is not a basis.

4. If \( f(x) = x^3 + 2x^2 + 3x + 4 \), then which of the following is true?
   - A. \( f(1) = 10 \)
   - B. \( f(-1) = -5 \)
   - C. \( f(0) = 4 \)

5. Consider the set \( T = \{v_1, v_2\} \) in \( H^2 \) then show that \( T \) is an orthonormal set.
   - a. Show that \( T \) is linearly independent.
   - b. Show that \( T \) is linearly dependent.

6. The Fourier transform of the triangle function
   \[ f(x) = \begin{cases} 0 & |x| > 1 \\ \frac{1}{2} & |x| < 1 \end{cases} \]
   is given by \( \hat{f}(\xi) = \frac{\sin(\xi)}{\xi} \).
   - a. Compute \( \hat{f}(0) \).
   - b. Compute \( \hat{f}(\infty) \).

7. Let \( T \) be the Frobenius transformation of the characteristic function
   \[ f(x) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases} \]
   then show \( \langle f, \phi \rangle = 0 \).
   - a. Compute \( \langle f, \phi \rangle \).
   - b. Compute \( \langle f, \phi \rangle \) when \( \phi(x) = 1 \).

8. Let \( f(x) \) be an \( L^2 \) function and \( \hat{f}(x) \) is the Fourier transform of \( f(x) \) then
   - a. If \( f(x) \) can be written as \( \int_{-\infty}^{\infty} d\xi \langle \phi, f(x) \rangle \) \( \phi(\xi) \), then show \( \langle \phi, f(x) \rangle \phi(\xi) \).
   - b. Show that \( \phi(\xi) \) may or may not be \( L^2 \) functions.

9. Consider the set \( T = \{(0,2,0), (x,0,y) | x, y \in \mathbb{R} \} \) then show that
   - a. \( T \) is linearly independent.
   - b. \( T \) is not a basis.
   - c. \( T \) is an orthonormal set.

10. Let \( \phi(x) \) be the Fourier transform of the characteristic function
    \[ f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \]
    then show \( \langle \phi, f(x) \rangle = 0 \).
    - a. Compute \( \langle \phi, f(x) \rangle \).
    - b. Compute \( \langle \phi, f(x) \rangle \) when \( \phi(x) = 1 \).