Unit 3 - Week 1

Assignment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each question. In case of multiple answers no partial marks will be awarded if all the correct choices are not selected. 0 marks for questions not attempted.

1) Items coming off a production line are marked defective (D) or non-defective (N). Items are observed and their condition noted. This is continued until two consecutive defectives are produced or five items have been checked, which ever occurs first. Then the sample space for this experiment is given by

\[ \Omega = \{DD, DNDD, DNDN, DNNDD, DNNND, DNNNN, NDDD, NDNDD, NNNDD, NNNND, NNNNN\} \]

No, the answer is incorrect.
Score: 0

Accepted Answers:
\[ \Omega = \{DD, DNDD, DNDN, DNNDD, DNNND, DNNNN, NDDD, NDNDD, NNNDD, NNNND, NNNNN, \]
\[ NDD, NNDN, NNND, NNND, NNNN, NNND, NNND, NNNN, NNNND, NNNND, NNNND, A \]

2) There are two urns \( U_1 \) and \( U_2 \). \( U_1 \) contains four white and four black balls, and \( U_2 \) is empty. Three balls are drawn at random from \( U_1 \) and transferred to \( U_2 \). Then one ball is drawn at random from each urn \( U_1 \) and \( U_2 \). Based on the given information, identify the TRUE statements.

The probability that ball drawn from \( U_1 \) is white is \( \frac{1}{2} \)

The probability that ball drawn from \( U_1 \) is black is \( \frac{1}{2} \)

The probability that ball drawn from \( U_2 \) is white is \( \frac{1}{2} \)

The probability that ball drawn from \( U_2 \) is black is \( \frac{1}{2} \)

No, the answer is incorrect.
Score: 0

Accepted Answers:
The probability that ball drawn from \( U_1 \) is white is \( \frac{1}{2} \)
The probability that ball drawn from \( U_2 \) is white is \( \frac{1}{2} \)

3) Let \((\Omega, \mathcal{F}, P)\) be a probability space. Let \( A \) and \( B \) be any event in this probability space. If \( \sigma \{ A, B \} \) denotes sigma field generated by the events \( A \) and \( B \), then, which of the following statements are TRUE?

\[ \sigma \{ A, B \} = \sigma \{ A \cap B, A^C \cap B, A \cap B^C, A^C \cap B^C \} \]

2 points
4. Four tennis players A, B, C, D have the probabilities of winning a tournament as \( P(A) = 0.25, P(B) = 0.25, P(C) = 0.35, P(D) = 0.15 \). Before the tournament, the player C is injured and withdraws. Then the new probabilities of winning the tournament for A, B and D is

\[
P(A) = \frac{5}{13}, P(B) = \frac{5}{13}, P(D) = \frac{3}{13}.
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[
\sigma(A, B) = \sigma(A \cap B, A \cup B, A^C \cap B^C)
\]

\[
\sigma(A, B) = \sigma(A \cap B, A^C \cap B \cap A^C \cap B^C)
\]

4 points

5. In a city with one hundred taxis, 1 is green and 99 are blue. A witness observes a hit-and-run case by a taxi at night and recalls that the taxi was green, so the police arrest the green taxi driver who was on duty that night. The driver proclaims his innocence and hires you to defend him in court. You hire a scientist to test the witness’s ability to distinguish green and white taxis in night. The data suggests that the witness sees green cars as green 97% of the time and blue cars as green 5% of the time. Then the probability that the witness is saying the truth is

\[
\frac{97}{100} \times \frac{97}{100} = \frac{9409}{10000}
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[
\omega \neq 1, x, y, z
\]

\[
x + y + z = 0
\]

\[
\omega x - 2, \omega y - 2, \omega z - 2 = 0
\]

6. Let \( \omega \) be a complex cube root of unity with \( \omega \neq 1 \). A fair die is thrown three times. If \( x, y, \) and \( z \) are the numbers obtained on the die, then the probability that \( \omega x^2 + \omega y^2 + \omega z^2 = 0 \) is equal to

\[
\frac{1}{3}
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[
\frac{1}{3}
\]

7. Suppose you got two sticks, the length of one is three times that of the smaller stick. Break the larger stick randomly into two pieces. Then, the probability that you can make a triangle using the three pieces is

\[
\frac{1}{3}
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:
8) Let $\Omega = \{a, b, c, d\}$ be a sample space. Then which of the following are sigma fields over $\Omega$. 2 points

<table>
<thead>
<tr>
<th>$\mathcal{F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${\emptyset, \Omega, {a}, {b}, {a, b}, {a, c, d}, {b, c, d}}$</td>
</tr>
<tr>
<td>${\emptyset, \Omega, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c}}$</td>
</tr>
<tr>
<td>${\emptyset, \Omega, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, c, d}}$</td>
</tr>
<tr>
<td>${\emptyset, \Omega, {a}, {b}, {a, b}, {a, c, d}, {b, c, d}}$</td>
</tr>
</tbody>
</table>

No, the answer is incorrect.

Score: 0

Accepted Answers:
- $\{\emptyset, \Omega, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}\}$
- $\{\emptyset, \Omega, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$
- $\{\emptyset, \Omega, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$
- $\{\emptyset, \Omega, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}\}$

9) Let $A$ and $B$ be two events. Then, which of the following statements are TRUE? 2 points

- If $P(A) = \frac{1}{2}$ and $P(B') = \frac{1}{3}$, then, $A$ and $B$ can be mutually exclusive events.
- The probability that exactly one of the events $A$ or $B$ occurs is equal to $P(A) + P(B) - 2P(A \cap B)$.
- If $A$ and $B$ are mutually exclusive events then $P(A)P(B) = 0$.
- If $A$ and $B$ are independent events such that $A \subseteq B$, then either $P(A) = 0$ or $P(B) = 1$.

No, the answer is incorrect.

Score: 0

Accepted Answers:
- The probability that exactly one of the events $A$ or $B$ occurs is equal to $P(A) + P(B) - 2P(A \cap B)$.
- If $A$ and $B$ are independent events such that $A \subseteq B$, then either $P(A) = 0$ or $P(B) = 1$.

10) An electronic assembly consists of two subsystems, say $A$ and $B$. From previous testing procedures, the following probabilities are assumed to be known: $P(A$ fails$)=0.20$, $P(A$ and $B$ both fail$)=0.15$, $P(B$ fails alone$)=0.1$. Identify the which of the following statements are TRUE. 0 points

<table>
<thead>
<tr>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(A$ fails$)$</td>
</tr>
<tr>
<td>$P(A$ fails alone$)$</td>
</tr>
<tr>
<td>$P(B$ fails alone$)$</td>
</tr>
<tr>
<td>$P(A$ fail$</td>
</tr>
<tr>
<td>$P(A$ fails alone$</td>
</tr>
<tr>
<td>$P(B$ fails alone$</td>
</tr>
</tbody>
</table>

No, the answer is incorrect.

Score: 0

Accepted Answers:
- $P(A$ fail$ | B has failed$)$
- $P(A$ fail$ | A or B fail$)$
- $P(B$ fail$ | A or B fail$)$