Assignment-8
[Solutions]

Sol. [1.] $\boxed{a)}$ Let λ_0 be the total arrival rate at queue.
Let λ_1 be the arrival rate at node F.

By Burke's theorem, we have

$$\lambda_0 = \lambda + \lambda_1 \;, \quad \lambda_1 = q\lambda_0$$

$$\Rightarrow \lambda_0 = \frac{\lambda}{p} \qquad \Rightarrow \rho = \frac{\lambda_0}{\mu}$$

Now, using Jackson's theorem, we have

$$P(N=i) = (1-\rho)\rho^i, \; i=0,1,2,\ldots$$
$$= \left(1-\frac{\lambda}{p\mu}\right)\frac{\lambda}{p\mu}, \; i=0,1,2,\ldots$$

Sol. [2.] $\boxed{d)}$

Sol. [3.] SPN are obtained by associating stochastic and timing information to Petri nets which is performed by attaching firing time to each transition.

$\boxed{c)}$

Sol. [4.] $\boxed{b)}$

Sol. [5.] $\boxed{a)}$

Using Burke's theorem, we have

at queue 1, $\lambda_1 = \lambda + q\lambda_2$,

at queue 2, $\lambda_2 = p\lambda_1$.

On solving the above two equations obtained, we have

$$\lambda_1 = \lambda + q\,p\lambda_1$$

$$\Rightarrow \lambda_1(1-qp) = \lambda$$

$$\Rightarrow \lambda_1 = \frac{\lambda}{1-qp}, \quad \lambda_2 = \frac{p\lambda}{1-qp}$$

$\therefore$, $\lambda_1 = 12.5$ /sec, $\lambda_2 = 6.25$ /sec

Sol [6.] a)

Using Burke's theorem

At queue 1, $\lambda_1 = q\lambda_2 + (1-p)\lambda_2$

$$\Rightarrow \lambda_2 = \frac{(1-q)}{(1-p)}\lambda_1$$

Sol. [7.] $\boxed{b)}$

$$P[\text{ Queue 1 is busy}] = P[\text{ Queue 1 has atleast 1 job}]$$
$$= 1 - P[\text{ No job at queue 1}]$$
$$= 1 - P[0, c]$$

Similarly,

$$P[\text{ Queue 2 is busy}] = 1 - P[c, 0]$$

Sol. [8.] $\boxed{a)}$