Week 7

(1)
$$P(T=k) = P(T_1 + T_2 = k)$$

T = total no. of customers in 2 systems

T_1 = no. of customers in system 1

T_2 = " 2.

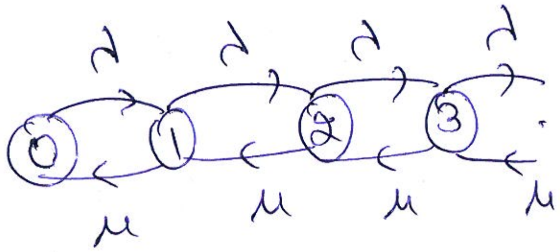
$$= \sum_{r=0}^{k} P(T_1 = r)\, P(T_2 = k-r)$$

$$= \sum_{r=0}^{k} \left[(1-\rho)\rho^{r}\right]\left[(1-\rho)\rho^{k-r}\right]$$

$$= (1-\rho)^2 \rho^k \sum_{r=0}^{k} 1 = (k+1)\rho^k (1-\rho)^2$$

option (c)

(2)



$\lambda = 4$ per hour

$\mu = 10$ min per customer $= 6$ customer per hour

$$\rho = 4/6 = 2/3$$

(a) average no. of customers in shop

$$= \frac{\rho}{1-\rho} = 2$$

(b) average no. of customers waiting

$$= \frac{\rho^2}{1-\rho} = \frac{4/9}{1/3} = \frac{4}{3}$$

(c) %age of time an arrival can walk right in without wait

$= P(\text{no customer in system})$

$= \pi_0 = 1-\rho = 1-\frac{2}{3} = \frac{1}{3}$

(d) Expected waiting time

$= \dfrac{\text{Expected no. waiting}}{\lambda}$ [by Little's law]

$= \dfrac{\rho^2}{\lambda(1-\rho)} = \dfrac{1}{4} \cdot \dfrac{4}{3} = \dfrac{1}{3}$

3) M/M/1 queue with $\lambda = 3$ per minute

Give

$P(\text{no. of customers} \leq 5) = 0.9$

$\Rightarrow \pi_0 + \pi_1 + \dots + \pi_5 = 0.9$

$(1-\rho) + (1-\rho)\rho^2 + \dots + (1-\rho)\rho^5 = 0.9$

$(1-\rho)[1 + \rho^2 + \dots + \rho^5] = 0.9$

$\dfrac{(1-\rho)1(1-\rho^6]}{1-\rho} = 0.9$

$\rho^6 = 0.1$

$\rho = 0.6812$

$\mu = \frac{\lambda}{\rho} = 4.4$

option (a)

(4) M|M|1 model with $\lambda = 5$ customer per hour

$\mu =$ 6 minutes per customer

$=$ 10 customers per hour.

(a) Expected waiting time

$$= \frac{1}{\lambda} \cdot \frac{\rho^2}{1-\rho} = \frac{\left(\frac{1}{2}\right)^2}{5\left(1-\frac{1}{2}\right)} = \underline{\underline{\frac{1}{10}}} \text{ hours}$$

(b) P(patient doesnot have to wait)

$$= \Pi_0 = 1-\rho = 1-\frac{1}{2} = \frac{1}{2} = \underline{\underline{0.5}}$$

(5) It is an M|M|1 queue

λ unknown

$\mu = \frac{1}{10}$ ms $= 0.1$ ms

P(idle buffer) = P(queue empty)

$$= \Pi_0 = 1-\rho = 0.8$$

$$\Rightarrow \rho = 0.2$$

$$\Rightarrow \frac{\lambda}{0.1} = 0.2$$

$$\Rightarrow \underline{\underline{\lambda = 0.02}}$$

$\therefore$ mean waiting time $= \frac{1}{\lambda} \frac{\rho^2}{1-\rho} = \frac{0.04}{(0.02)(0.8)} = \underline{\underline{2.5 \text{ ms}}}$