Stochastic Processes
Assignment (Week 12)

1. Let \( \{X(t), t \geq 0\} \) be a stochastic process. Which of the following is always TRUE?
   (a) If \( \{X(t), t \geq 0\} \) is a second-order process then it must be a wide-sense stationary process.
   (b) If \( \{X(t), t \geq 0\} \) is a wide sense stationary process then it must be a second order process.
   (c) If \( \{X(t), t \geq 0\} \) is a second order process with constant mean then it must be a wide-sense stationary process.
   (d) \( \{X(t), t \geq 0\} \) always has independent increments.

2. Consider the process \( \{X(t), t \geq 0\} \), where
   \[ X(t) = Acos(\theta t) + Bsin(\theta t), \]
   where \( A \) and \( B \) are known to be uncorrelated random variables each with mean and variance as 0 and \( \sigma^2 \) respectively and \( \theta \) is a constant.
   (a) \( E(X(t)) = 0 \)
   (b) \( Cov(X(t), X(s)) \) is not a function of \( |t - s| \)
   (c) \( E(X(t)^2) = \sigma^2 \)
   (d) \( \{X(t), t \geq 0\} \) is a wide-sense stationary process.

3. Let \( Z_1 \) and \( Z_2 \) be two independent normally distributed random variables, each having mean 0 and variance \( \sigma^2 \). Let \( \lambda \in \mathbb{R} \). Define \( X_t = Z_1 \cos \lambda t + Z_2 \sin \lambda t \). Which of the following is not TRUE?
   (a) \( \{X(t), t \geq 0\} \) is a second order process.
   (b) \( E(X(t)) = 0 \).
   (c) \( E(X(t)^2) \)
   (d) \( \{X(t), t \geq 0\} \) is a wide sense stationary process.

4. In a communication system, a carrier signal at a receiver is modeled as a stochastic process \( \{X(t) = \cos(2\pi ft + \theta); t \geq 0\} \) where \( \theta \in [-\pi, \pi] \) and \( f \) is a constant.
   (a) \( \{X(t), t \geq 0\} \) is a second order process.
   (b) \( E(X(t)) = 0 \).
   (c) \( Cov(X(t), X(s)) \) is a function of \( |t - s| \)
   (d) \( \{X(t), t \geq 0\} \) is a wide sense stationary process.

5. Let \( \{X(t), t \geq 0\} \) be a strict sense stationary stochastic process. Let \( A \) be a positive random variable independent of the stochastic process \( \{X(t), t \geq 0\} \). Define
   \[ Y(t) = AX(t) \]
   Then, which of the following is TRUE?
   (a) \( \{Y(t), t \geq 0\} \) is always a strict sense stationary process.
   (b) \( \{Y(t), t \geq 0\} \) is never a strict sense stationary process.
   (c) \( \{Y(t), t \geq 0\} \) may or may not be a strict sense stationary process.
   (d) \( \{Y(t), t \geq 0\} \) is not even a stochastic process.
6. In a communication system, the carrier signal at the receiver is modeled by \( Y(t) = X(t) \cos(2\pi wt + \Theta) \) where \( \{X(t), t \geq 0\} \) is a zero-mean and wide-sense stationary process, \( \Theta \) is a uniform distributed random variable with interval \((-\pi, \pi)\) and \( w \) is a positive constant. Assume that, \( \Theta \) is independent of the process \( \{X(t), t \geq 0\} \). Then, which of the following is not TRUE?

(a) mean function of \( \{Y(t), t \geq 0\} \) is independent of \( t \).
(b) \( E(Y(t)^2) \) is finite.
(c) covariance function of \( \{Y(t), t \geq 0\} \) is \( 0.5 \cos(2\pi \omega(t - s)) \text{Cov}(X(t), X(s)) \)
(d) \( \{Y(t), t \geq 0\} \) is not a wide-sense stationary process.

7. Let \( \{X(t), t \geq 0\} \) be a stochastic process with independent increments. Then, which of the following is always TRUE?

(a) \( \{X(t), t \geq 0\} \) is a Markov process.
(b) \( \{X(t), t \geq 0\} \) need not be a Markov process.
(c) \( \{X(t), t \geq 0\} \) is a wide-sense stationary stochastic process.
(d) \( \{X(t), t \geq 0\} \) is a strict-sense stationary process.

8. Let \( \{N(t), t \geq 0\} \) be a Poisson process with parameter \( \lambda \). Then, which of the following is not TRUE?

(a) \( \{N(t), t \geq 0\} \) is a Markov process.
(b) \( \{N(t), t \geq 0\} \) is a wide-sense stationary process.
(c) \( \{N(t), t \geq 0\} \) has independent increments.
(d) \( E(N(t)^2) < \infty \).

9. Let \( \{N(t), t \geq 0\} \) be a Poisson process with parameter \( \lambda \). Consider the process \( \{X(t), t \geq 0\} \) where

\[
X(t) = N(t + L) - N(t)
\]

where \( L \) is a positive constant. Then,

(a) \( E(X(t)) = 0 \)
(b) \( \text{Cov}(X(t), X(s)) \) is not a function of \( |t - s| \)
(c) \( E(X(t)^2) = \sigma^2 \)
(d) \( \{X(t), t \geq 0\} \) is a wide-sense stationary process.

10. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has \( k \) offsprings is \( p_k \), where \( p_0 = \frac{1}{5}, p_1 = \frac{3}{5} \) and \( p_2 = \frac{1}{5} \). Assume that particles act independently and identically irrespectively of the generation. The probability of extinction equals

(A) \( \frac{1}{5} \)  \( \frac{3}{5} \)  \( \frac{2}{5} \)  \( \frac{1}{5} \)  \( D \) 1

11. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has \( k \) offsprings is \( p_k \), where \( p_0 = 0.5, p_1 = 0.2, p_2 = 0.3 \). Assume that particles act independently and identically irrespectively of the generation. The probability that there is at least one particle in first generation equals

(A) 0.5  \( \frac{1}{4} \)  \( \frac{3}{4} \)  \( \frac{1}{4} \)  \( D \) 1

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