Stochastic Processes
Assignment (Week 12)

1. Let $\{X(t), t \geq 0\}$ be a stochastic process. Which of the following is always TRUE?
   (a) If $\{X(t), t \geq 0\}$ is a second-order process then it must be a wide-sense stationary process.
   (b) If $\{X(t), t \geq 0\}$ is a wide sense stationary process then it must be a second order process.
   (c) If $\{X(t), t \geq 0\}$ is a second order process with constant mean then it must be a wide-sense stationary process.
   (d) $\{X(t), t \geq 0\}$ always has independent increments.

2. Consider the process $\{X(t), t \geq 0\}$, where $X(t)$ is defined as:
$$X(t) = Acos(\theta t) + Bsin(\theta t),$$
where A and B are known to be uncorrelated random variables each with mean and variance as 0 and σ^2 respectively and θ is a constant.
   (a) $E(X(t)) = 0$
   (b) $Cov(X(t), X(s))$ is not a function of $|t - s|$
   (c) $E(X(t)^2) = \sigma^2$
   (d) $\{X(t), t \geq 0\}$ is a wide-sense stationary process.

3. Let Z_1 and Z_2 be two independent normally distributed random variables, each having mean 0 and variance σ^2. Let $\lambda \in \mathbb{R}$. Define $X_t = Z_1 \cos \lambda t + Z_2 \sin \lambda t$. Which of the following is not TRUE?
   (a) $\{X(t), t \geq 0\}$ is a second order process.
   (b) $E(X(t)) = 0$.
   (c) $E(X(t)^2)$
   (d) $\{X(t), t \geq 0\}$ is a wide sense stationary process.

4. In a communication system, a carrier signal at a receiver is modeled as a stochastic process $\{X(t) = cos(2\pi ft + \theta); t \geq 0\}$ where θ $U[-\pi, \pi]$ and f is a constant.
   (a) $\{X(t), t \geq 0\}$ is a second order process.
   (b) $E(X(t)) = 0$.
   (c) $Cov(X(t), X(s))$ is a function of $|t - s|$
   (d) $\{X(t), t \geq 0\}$ is a wide sense stationary process.

5. Let $\{X(t), t \geq 0\}$ be a strict sense stationary stochastic process. Let A be a positive random variable independent of the stochastic process $\{X(t), t \geq 0\}$. Define
$$Y(t) = AX(t)$$
Then, which of the following is TRUE?
   (a) $\{Y(t), t \geq 0\}$ is always a strict sense stationary process.
   (b) $\{Y(t), t \geq 0\}$ is never a strict sense stationary process.
   (c) $\{Y(t), t \geq 0\}$ may or may not be a strict sense stationary process.
   (d) $\{Y(t), t \geq 0\}$ is not even a stochastic process.

6. In a communication system, the carrier signal at the receiver is modeled by $Y(t) = X(t)\cos(2\pi wt + \Theta)$ where $\{X(t), t \geq 0\}$ is a zero-mean and wide-sense stationary process, Θ is a uniform distributed random variable with interval $(-\pi, \pi)$ and w is a positive constant. Assume that, Θ is independent of the process $\{X(t), t \geq 0\}$. Then, which of the following is not TRUE?

   (a) mean function of $\{Y(t), t \geq 0\}$ is independent of t.

   (b) $E(Y(t)^2)$ is finite.

   (c) covariance function of $\{Y(t), t \geq 0\}$ is $0.5\cos(2\pi\omega(t-s))Cov(X(t), X(s))$

   (d) $\{Y(t), t \geq 0\}$ is not a wide-sense stationary process.

7. Let $\{X(t), t \geq 0\}$ be a stochastic process with independent increments. Then, which of the following is always TRUE?

   (a) $\{X(t), t \geq 0\}$ is a Markov process.

   (b) $\{X(t), t \geq 0\}$ need not be a Markov process.

   (c) $\{X(t), t \geq 0\}$ is a wide-sense stationary stochastic process.

   (d) $\{X(t), t \geq 0\}$ is a strict-sense stationary process.

8. Let $\{N(t), t \geq 0\}$ be a Poison process with parameter λ. Then, which of the following is not TRUE?

   (a) $\{N(t), t \geq 0\}$ is a Markov process.

   (b) $\{N(t), t \geq 0\}$ is a wide-sense stationary process.

   (c) $\{N(t), t \geq 0\}$ has independent increments.

   (d) $E(N(t)^2) < \infty$.

9. Let $\{N(t), t \geq 0\}$ be a Poison process with parameter λ. Consider the process $\{X(t), t \geq 0\}$ where

$$X(t) = N(t+L) - N(t)$$

   where L is a positive constant. Then,

   (a) $E(X(t)) = 0$

   (b) $Cov(X(t), X(s))$ is not a function of $|t-s|$

   (c) $E(X(t)^2) = \sigma^2$

   (d) $\{X(t), t \geq 0\}$ is a wide-sense stationary process.

10. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has k offsprings is p_k, where $p_0 = \frac{1}{5}, p_1 = \frac{3}{5}$ and $p_2 = \frac{1}{5}$. Assume that particles act independently and identically irrespectively of the generation. The probability of extinction equals
(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) 1

11. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has k offsprings is p_k, where $p_0 = 0.5, p_1 = 0.2, p_2 = 0.3$. Assume that particles act independently and identically irrespectively of the generation. The probability that there is at least one particle in first generation equals
(A) 0.5 (B) 0.25 (C) 0.75 (D) 1