Stochastic Processes
Assignment (Week 11)

1. Boys arrive in a queue according to a Poisson process with rate λ_1 and girls arrive in the same queue according to a Poisson process with rate λ_2 independently of the arrival of boys. The probability that first arrival in the queue is a girl is

   (a) $\frac{\lambda_1}{\lambda_1+\lambda_2}$

   (b) $\frac{\lambda_2}{\lambda_1+\lambda_2}$

   (c) $1-\frac{1}{\lambda_1+\lambda_2}$

   (d) $\frac{1}{\lambda_1+\lambda_2}$

2. Let $\{X(t)\}$ and $\{Y(t)\}$ be two independent Poisson processes with rates λ_1 and λ_2 respectively. Let $Z(t) = X(t) + Y(t)$. Then

   (a) $Z(t)$ is not a Poisson process.

   (b) $Z(t)$ is a Poisson process with rate $\lambda_1 + \lambda_2$.

   (c) $Z(t)$ is a Poisson process with rate $\min \lambda_1, \lambda_2$.

   (d) $Z(t)$ is a Poisson process with rate $\lambda_1\lambda_2$.

3. Let mean function $m(t)$ of a renewal process $\{X(t), t \geq 0\}$ be $3t$. Then, $P(X(10) = 0)$ is equal to

   (a) e^{-30}

   (b) e^{-10}

   (c) $e^{-\frac{10}{3}}$

   (d) $e^{-\frac{3}{10}}$

4. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ. Then, its mean function $m(t)$ is

   (a) a constant function of t.

   (b) a linear function of t.

   (c) a quadratic function of t.

   (d) a cubic function of t.

5. Let $\{X_1, X_2, \ldots\}$ be a sequence of i.i.d. non-negative random variables. Define

   $$S_0 = 0,\ S_n = \sum_{i=1}^{n} X_i,\ n = 1, 2, \ldots$$

   Let $N(t)$ be defined as

   $$N(t) = max\{n : S_n \leq t\}, t \geq 0$$

   Then, which of the following is TRUE?

   (a) $N(t) \geq n \Longleftrightarrow S_n \leq t$

   (b) $N(t) \geq n \Longleftrightarrow S_n \geq t$

   (c) $N(t) \leq n \Longleftrightarrow S_n \geq t$

   (d) $N(t) \leq n \Longleftrightarrow S_n \leq t$

6. Let $\{N(t), t \geq 0\}$ be a renewal process with inter-arrival distribution $U(0, 1)$, i.e., $X_i \sim U(0, 1)$. Let $m(t)$ denote the renewal function of $\{N(t), t \geq 0\}$. Then, which of the following is TRUE for $t \in [0, 1]$?

(a) $m(t) = e^t$

(b) $m(t) = e^{-t}$

(c) $m(t) = e^t - 1$

(d) $m(t) = e^{-t} + 1$

7. Consider an age replacement model. Let X be the lifetime of a component with cumulative distribution function F. The component is replaced by a new one upon failure or by time T whichever comes first. The cost of new component is c_1 and cost incurred for replacement is c_2. Let $N(t)$ and $R(t)$ represents the number of components replaced and total cost incurred by time t. Note that $\{N(t), t \geq 0\}$ represent the renewal process and $\{R(t), t \geq 0\}$ is reward renewal process. Then, long run average cost is given by

   (a) $\frac{E(X)}{E(R)}$

   (b) $\frac{E(N)}{E(X)}$

   (c) $\frac{E(R)}{E(N)}$

   (d) $\frac{E(R)}{E(X)}$

8. A photographer has a camera that works on a single battery. As soon as the battery in use fails, he immediately replaces it with a new battery. If the lifetime of a battery (in hours) follows uniform distribution $U(1, 25)$, then at what rate does photographer has to change batteries?

   (a) $\frac{1}{13}$

   (b) 13

   (c) $\frac{1}{25}$

   (d) 25

9. Let $f(x) = \lambda^2 x e^{-\lambda x}$ be a probability density function of lifetime variable X. The corresponding renewal function is

   (a) $\frac{\lambda t}{2}$

   (b) $\frac{1-e^{-2\lambda t}}{4}$

   (c) $\frac{\lambda t}{2} - \frac{1-e^{-2\lambda t}}{4}$

   (d) $\frac{\lambda t}{2} + \frac{1-e^{-2\lambda t}}{4}$

10. Let $\{N(t), t \geq 0\}$ be a Poison process with parameter λ. Let X_1 be time of occurrence of first event. Then, distribution of X_1 is

   (a) Geometric distribution

   (b) Exponential distribution

   (c) Bernoulli distribution

   (d) Uniform distribution.