Stochastic Processes
Assignment (Week 10)

1. Let $\{W(t),\ t \geq 0\}$ be a Wiener process. Which of the following is TRUE?
   (a) $Cov(W(t), W(s)) = min\{s, t\}$
   (b) $Cov(W(t), W(s)) = (t - s)$
   (c) $Cov(W(t), W(s)) = s$
   (d) $Cov(W(t), W(s)) = t$

2. Suppose that $\{S(t), t \geq 0\}$, is a geometric Brownian motion with drift parameter $\mu = 0.01$ and volatility parameter $\sigma = 0.2$. If $S(0) = 100$. Then, $E[S(10)]$ is
   (a) 103.04
   (b) 105.00
   (c) 102.12
   (d) 100.01

3. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Then, $E((W(t) - W(s))^2)$ is equal to
   (a) $(t - s)^2$
   (b) $4(t - s)^3$
   (c) $(t - s)$
   (d) 0

4. Let $\{S(t), t \geq 0\}$ be a geometric Brownian motion with $S(0) = a$. Then, $E(S(t))$ is equal to
   (a) $ae^{\mu + \frac{\sigma^2}{2}}$
   (b) $a\mu$
   (c) ae^{μ}
   (d) μ

5. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Then, $E(W(s)|W(t) = x)$ for $0 < s < t$ is equal to
   (a) $\frac{sx}{t}$
   (b) $\frac{st}{x}$
   (c) $\frac{tx}{s}$
   (d) $\frac{x}{st}$

6. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Then, $Cov(W(s), W(t))$ for $0 < s < t$ is equal to
   (a) s
   (b) t
   (c) 0
   (d) $t - s$

7. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Then, $\{W(t)^2, t \geq 0\}$ is
   (a) martingale
   (b) sub-martingale
   (c) super-martingale

(d) not a martingale.

8. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Then, $E(W(t)|W(s) = x)$ for $0 < s < t$ is equal to

(a) $x + t - s$

(b) $x + t$

(c) $x - s$

(d) x