Stochastic Processes
Assignment (Week 9)

1. Let $\{N(t),\ t \geq 0\}$ be a Poisson process with rate 1. Which of the following stochastic process is martingale with respect to the natural filtration?

   (a) $\{N(t) - t, t \geq 0\}$

   (b) $\{N(t)^2 - t, t \geq 0\}$

   (c) $\{(N(t) - t)^2 - t, t \geq 0\}$

   (d) $\{N(t),\ t \geq 0\}$

2. Which of the following statements is TRUE?

   (a) The mean of a martingale process is constant with respect to time.

   (b) The mean of a martingale process is time varying.

   (c) The mean of a martingale process need not exist.

   (d) Every stochastic process is a martingale.

3. Which of the following statements about a stochastic process $\{X(t),\ t \geq 0\}$ is FALSE?

   (a) The process $\{X(t),\ t \geq 0\}$ is sub-martingale if and only if the process $\{-X(t),\ t \geq 0\}$ is super-martingale.

   (b) The process $\{X(t),\ t \geq 0\}$ is a martingale implies that the process $\{X(t),\ t \geq 0\}$ is sub-martingale.

   (c) The process $\{X(t),\ t \geq 0\}$ is a martingale if and only if the process $\{X(t),\ t \geq 0\}$ is both a sub-martingale and a super-martingale.

   (d) The process $\{X(t),\ t \geq 0\}$ is a martingale if and only if it is Markov process.

4. If $\{W(t), t \geq 0\}$ is a Brownian motion, then which of the following is NOT a martingale?

   (a) $\{W(t)^2 - t, t \geq 0\}$

   (b) $\{W(t), t \geq 0\}$

   (c) $\{W(t) - t, t \geq 0\}$

   (d) $\{W(t)^2, t \geq 0\}$

5. For what values of c, the process $S(t) = c(\sigma+1)^{N(t)}$, $\sigma > -1$ is a constant, is a martingale where $\{N(t), t \geq 0\}$ is a Poisson process with rate λ.

   (a) $e^{-\lambda\sigma t}$

   (b) $e^{\lambda\sigma t}$

   (c) σ

   (d) λ

6. Let X_n be a symmetric random walk and $\mathbb{F}_n$ be a filtration. The stochastic process

$$Y_n = (-1)^n cos(\pi X_n)$$

   is

   (a) a martingale with respect to $\mathbb{F}_n$.

   (b) a sub-martingale with respect to $\mathbb{F}_n$ but not martingale..

   (c) a super-martingale with respect to $\mathbb{F}_n$ but not martingale.

   (d) not a martingale with respect to $\mathbb{F}_n$.

7. Let $\{W(t), t \geq 0\}$ be a Wiener process. The stochastic process $exp(\sigma W(t) - \frac{\sigma^2}{2}t)$, where σ is a positive constant

   (a) a martingale with respect to $\mathbb{F}_n$.

   (b) a sub-martingale with respect to $\mathbb{F}_n$ but not martingale..

   (c) a super-martingale with respect to $\mathbb{F}_n$ but not martingale.

   (d) not a martingale with respect to $\mathbb{F}_n$.

8. Let X be a random variable and $\mathbb{G}_1$ and $\mathbb{G}_2$ be two sub-σ-fields of $\mathbb{F}$ such that $\mathbb{G}_1 \subseteq \mathbb{G}_2$. Which of the following statements is FALSE?

   (a) $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(X \mid \mathbb{G}_1)$.

   (b) $E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1) = E(X \mid \mathbb{G}_1)$.

   (c) $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(X \mid \mathbb{G}_2)$.

   (d) $E(E(X \mid \mathbb{G}_1) \mid \mathbb{G}_2) = E(E(X \mid \mathbb{G}_2) \mid \mathbb{G}_1)$.

9. Let X be a random variable and let $\mathbb{F} = [\emptyset, \Omega]$ be the σ-field. Then,

   (a) $E(X \mid \mathbb{F})$ need not exist.

   (b) $E(X \mid \mathbb{F})$ always exists.

   (c) X must be a degenerate random variable.

   (d) $E(X \mid \mathbb{F})$ always exists and is same as value of X.

10. For what values of c, the process $S(t) = e^{W(t)-ct}$ is a martingale where $\{W(t), t \geq 0\}$ is a Wiener process.

   (a) -0.5

   (b) 0.5

   (c) 1

   (d) -1

11. Consider the process $S(t) = \mu t + \sigma W(t)$ where $\{W(t), t \geq 0\}$ is a Wiener process. Which of the following is FALSE?

   (a) $S(t)$ is a martingale for every value of μ.

   (b) $S(t)$ is a martingale only if $\mu = 0$.

   (c) $S(t)$ is a sub-martingale for $\mu > 0$.

   (d) $S(t)$ is a super-martingale for $\mu < 0$.