Stochastic Process
Assignment (Week 7)

1. Consider that two identical $M/M/1$ queueing systems with the same rates λ and μ, which are in operation side by side (with separate queues) in a premises. The probability that there are a total of k number of customers in the two systems taken together in long-run is given by

   (A) $(k)\rho^{k+1}(1-\rho)^2$ (B) $(k+1)\rho^k(1-\rho)$
   (C) $(k+1)\rho^k(1-\rho)^2$ (D) $(k)\rho^k(1-\rho)^2$

2. Mr. Rajesh runs a one-person, unisex hair salon. He finds that customers seem to arrive according to a Poisson process with a mean arrival rate of 4 per hour. Because of his excellent reputation, customers were always willing to wait. The data further showed that customer processing time (aggregated female and male) was exponentially distributed with an average of 10 min.

   (a) The average number of customers in the shop is

   (A) 1 (B) 2 (C) 3 (D) 4

   (b) The average number of customers waiting for a haircut

   (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) 1

   (c) The percentage of time an arrival can walk right in without having to wait at all

   (A) 0.166 (B) 0.706 (C) 0.223 (D) 0.333

   (d) The expected waiting time of a customer is

   (A) 0.166 (B) 0.706 (C) 0.223 (D) 0.333

3. Consider a $M/M/1$ queueing model with arrival rate λ and service rate μ. The service rate where customers arrive at a rate of 3 per minute, given that 90% of the time the queue contains less than or equal to 5 customers is given by

   (A) 4.4 (B) 4.8 (C) 2.2 (D) 3.9

4. Patients visit a doctor in accordance with a Poisson process at the rate of 5 per hour, and the time doctor takes to examine any patient is exponential with mean 6 minutes. All arriving patients attended by the doctor.

   (a) The expected waiting time of any patient who visits the doctor is given by

   (A) $\frac{1}{10}$ hours (B) $\frac{1}{2}$ hours (C) $\frac{1}{8}$ hours (D) $\frac{1}{4}$ hours

   (b) The probability that a patient does not have to wait on arrival

   (A) 0.5 (B) 0.25 (C) 0.75 (D) 0.33

5. Consider a multiplexer that collects traffic formed by messages arriving according to exponential distributed inter-arrival times. The multiplexer is formed by a buffer and a transmission line. Assume that, the transmission time of a message is exponential distributed with the mean value 10 ms. From measurements on the state of the buffer, we know that the idle buffer probability is 0.8. The mean delay (waiting time) for the message is

   (A) 32.87 ms (B) 10 ms (C) 8.0912 ms (D) 4.768 ms