

## Stochastic Processes Assignment (Week 4)

1. Let  $\{X_n, n = 1, 2, \dots\}$  be a Markov chain with state space  $S = \{1, 2, 3, \dots, 11\}$  such that one-step transition probabilities are given by

$$p_{ij} > 0 \text{ if } |i - j| \text{ is even}$$

$$p_{ij} = 0 \text{ if } |i - j| \text{ is odd}$$

Which of the following is TRUE?

- (a) Chain is irreducible.
  - (b) Chain is not aperiodic.
  - (c) State 5 is transient.
  - (d) State 2 is recurrent.
2. Let  $\{X_n, n = 1, 2, \dots\}$  be a Markov chain with state space  $S = \{1, 2, 3, \dots, 21\}$  such that one-step transition probabilities are given by

$$p_{ij} > 0 \text{ if } |i - j| \text{ is even}$$

$$p_{ij} = 0 \text{ if } |i - j| \text{ is odd}$$

Let  $p_{14} > 0$  and  $p_{25}^{(3)} > 0$ . Which of the following is Not TRUE?

- (a) Chain is irreducible.
  - (b) Chain is aperiodic.
  - (c) State 21 is transient.
  - (d) State 1 is recurrent.
3. Let  $X_0$  be an integer-valued random variable,  $P(X_0 = 0) = 1$ , that is independent of the i.i.d. sequence  $Z_1, Z_2, \dots$ , where  $Z_n$  can take values in the set  $\{-1, 1\}$  such that  $P(Z_n = -1) = p, P(Z_n = 1) = q$ . Let  $X_n = X_{n-1} + Z_n, n = 1, 2, \dots$ . The value of the probability  $P(X_5 = 3 | X_3 = 3)$  is equal to  
 (a)  $pq$     (b)  $2pq$     (c)  $p^2$     (d)  $q^2$

4. Consider a Markov chain with state space  $\{0, 1, 2\}$  and transition matrix  $P = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$  Which of the following is FALSE?
- (a) The chain is reducible.
  - (b) All states are positive recurrent.
  - (c) The chain is aperiodic.
  - (d) State 0 is transient state.

5. Consider a Markov chain with state space  $\{0, 1, 2, 3\}$  and transition matrix  $P = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$  Which of the following is true?
- (a) Markov chain is irreducible.
  - (b) Markov chain is not aperiodic.
  - (c) All states are recurrent.
  - (d) Markov chain consists of one closed communicating class given by  $\{0, 1\}$ .

6. The transition probability matrix of a discrete time Markov chain  $\{X_n, n = 0, 1, \dots\}$  having three states 1, 2 and 3 is  $P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$  and the initial distribution is  $\pi = (0, 1, 0)$ . The value of the probability  $P(X_{121} = 3, X_{152} = 1)$  is equal to  
 (a) 0.22    (b) 0.5    (c) 1    (d) 0

7. Consider a Markov chain with state space  $\{0, 1, 2, 3, 4\}$  and transition matrix  $P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}$

Which of the following is true?

- (a) All states are recurrent.  
 (b) States 0, 3, 4 are recurrent and states 2, 4 are transient.  
 (c) States 0, 1, 3, 4 are recurrent and state 2 is transient.  
 (d) Only states 3, 4 are recurrent.
8. Consider a Markov chain  $\{X_n, n = 1, 2, \dots\}$  with state space  $S = \{0, 1, 2, 3, 4\}$  and transition probability

matrix  $P$  given by  $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

Which of the following is true?

- (a) All states are recurrent.  
 (b) States 0, 1, 2, 3 are recurrent and state 4 is transient.  
 (c) States 0, 1, 2, 4 are transient and state 3 is recurrent.  
 (d) Only state 3 is recurrent.