Stochastic Processes
Assignment (Week 3)

1. Classify the following stochastic process based on the state space and index set. The number of customers in queue in front of an ATM for at the end of each hour of a day.

   (a) Discrete time discrete state stochastic process.
   (b) Discrete time continuous state stochastic process.
   (c) Continuous time discrete state stochastic process.
   (d) Continuous time continuous state stochastic process.

2. Classify the following stochastic process based on the state space and index set. Number of vehicles in parking of a shopping mall at any time during the day.

   (a) Discrete time discrete state stochastic process.
   (b) Discrete time continuous state stochastic process.
   (c) Continuous time discrete state stochastic process.
   (d) Continuous time continuous state stochastic process.

3. Classify the following stochastic process based on the state space and index set. The exchange rate of rupee into dollar at any time t.

   (a) Discrete time discrete state stochastic process.
   (b) Discrete time continuous state stochastic process.
   (c) Continuous time discrete state stochastic process.
   (d) Continuous time continuous state stochastic process.

4. Classify the following stochastic process based on the state space and index set. The number of particles emitted by a certain radioactive material undergoing radioactive decay during a certain period. .

   (a) Discrete time discrete state stochastic process.
   (b) Discrete time continuous state stochastic process.
   (c) Continuous time discrete state stochastic process.
   (d) Continuous time continuous state stochastic process.

5. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate 2. The value of $P(N(1.7) = 10, N(2.3) = 19, N(4.1) = 31)$ is equal to

   (a) $\frac{e^{-2*1.8}*(2*1.8)^{12}}{12!} * \frac{e^{-2*0.6}*(2*0.6)^{9}}{9!} \times \frac{e^{-2*1.7}*(2*1.7)^{10}}{10!}$
   (b) $\frac{e^{-2*4.1}*(2*4.1)^{31}}{31!} * \frac{e^{-2*2.3}*(2*2.3)^{19}}{19!} \times \frac{e^{-2*1.7}*(2*1.7)^{10}}{10!}$
   (c) $\frac{e^{-2*1.7}*(2*1.7)^{9}}{9!} * \frac{e^{-2*0.6}*(2*0.6)^{12}}{12!} \times \frac{e^{-2*1.8}*(2*1.8)^{31}}{31!}$
   (d) $\frac{e^{-2*2.3}*(2*2.3)^{19}}{19!} * \frac{e^{-2*0.6}*(2*0.6)^{9}}{9!} \times \frac{e^{-2*1.8}*(2*1.8)^{12}}{12!}$

6. Let $\{X_n, n = 0, 1, 2, \}$ be a DTMC with state space $\Omega = \{0, 1\}$ and one-step transition probability matrix $P = \begin{pmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{pmatrix}$ Let initial distribution be $\pi = (0.5, 0.5)$. Then, the value of the probability $P(X(3) = 0)$ is equal to
   (a) 1 (b) 0.5 (c) 0.26 (d) 0.33

7. Let $\{X_n, n = 0, 1, 2, \}$ be a DTMC with state space $\Omega = \{0, 1, 2, 3, 4, 5\}$ and one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.75 & 0.25 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$ Which of the following is NOT TRUE?
(a) Chain is reducible. (b) period of state 1 is 2 (c) $C(2) = \{2, 3\}$ (d) period of state 2 is 1.

8. Consider a simple symmetric random walk model. Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with

$$P(X_1 = 1) = 0.5 \text{ and } P(X_1 = -1) = 0.5.$$

Define $S_n = \sum_{i=1}^{n} X_i$. The value of the probability $P(X_6 = 5 \mid X_0 = 1)$ is equal to
(a) $\frac{6}{2^6}$ (b) $\frac{1}{2^6}$ (c) 0 (d) $\frac{5}{2^6}$

9. Consider a simple symmetric random walk model. Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with

$$P(X_1 = 1) = 0.5 \text{ and } P(X_1 = -1) = 0.5.$$

Define $S_n = \sum_{i=1}^{n} X_i$. The value of the probability $P(X_6 = 4 \mid X_0 = 1)$ is equal to
(a) $\frac{6}{2^6}$ (b) $\frac{1}{2^6}$ (c) 0 (d) $\frac{5}{2^6}$