Stochastic Process
Assignment (Week 1)

1. Let E and F be two events such that $P(E) = 0.7, P(F) = 0.4$ and $P(E \bigcap F^c) = 0.4$. Then, $P(F \mid E \bigcup F^c)$ is equal to
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

2. Let joint density function of (X, Y) be $f(x, y) = c(x + y)$ if $-x < y < x, 0 < x < 1$ and 0 otherwise. Then value of c is equal to

   (a) $\frac{2}{3}$ (b) 3 (c) $\frac{3}{2}$ (d) 1

3. Let X be a continuous random variable with probability density function $f(x)$ satisfying

   $$f(x) = 2\phi(x)\Phi(x), \ x \in \mathbb{R}$$

   where $\phi(x)$ and $\Phi(x)$ represent pdf and CDF of standard normal random variable respectively.
   (a) $E(X) > 0$ (b) $E(X) < 0$ (c) $P(X \geq 0) < 0.25$ (d) $P(X \leq 0) > 0.5$

4. A circle of random radius R (in cm) is constructed. Let $R \sim U(0, 1)$. The probability that area of the circle is less than 1 cm^2 is
   (a) $\frac{1}{4\sqrt{\pi}}$ (b) $\frac{1}{3\sqrt{\pi}}$ (c) $\frac{1}{2\sqrt{\pi}}$ (d) $\frac{1}{\sqrt{\pi}}$

5. A system consisting of n components function if and only if atleast one of n components function. Suppose that all n components function independently, each with probability $\frac{3}{4}$. If reliability of the system is $\frac{63}{64}$, then the value of n is
   (a) 2 (b) 4 (c) 3 (d) 5

6. There are two urns U_1 and U_2. U_1 contains four white and four black balls, and U_2 is empty. Four balls are drawn at random from U_1 and transferred to U_2. Then a ball is drawn at random from U_2. The probability that ball drawn from U_2 is white is
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

7. Let w be a complex cube root of unity with $w \neq 1$. A fair die is thrown three times. If x, y and z are the numbers obtained on the die. The probability that $w^x + w^y + w^z = 0$ is
   (a) $\frac{1}{9}$ (b) $\frac{1}{2}$ (c) $\frac{2}{9}$ (d) $\frac{2}{3}$

8. The coefficients a, b and c of the quadratic equation $ax^2 + bx + c = 0$ are determined by rolling a fair die three times in a row. The probability that both the roots of the equation are real is
   (a) $\frac{43}{216}$ (b) $\frac{173}{216}$ (c) $\frac{73}{216}$ (d) $\frac{143}{216}$