Week 3

1. The stochastic process is to observe number of customers in front of ATM at end of each hour of a day.

   It is a discrete time discrete ~~space~~ state stochastic process.

2. The stochastic process is number of vehicles in parking of a shopping mall at any time during the day.

   It is a continous time discrete state stochastic process.

3. The stochastic process is exchange rate of Rupee into dollar at any time t.

   It is a continuous time continuous state stochastic process.

4. The stochastic process is no. of particles emitted by a certain radioactive material undergoing radioactive decay during a certain period.

   It is a continous time discrete state stochastic process.

$\lambda = 2$

(5) $P(N(1.7)=10,\ N(2.3)=19,\ N(4.1)=31)$

$= P(N(1.7)=10,\ N(2.3)-N(1.7)=9,\ N(4.1)-N(2.3)=12)$

$\because$ N(t) has independent increments

$= P(N(1.7)=10)\ P(N(2.3)-N(1.7)=9)\ P(N(4.1)-N(2.3)=12)$

$$= e^{-2(1.7)}\frac{(2(1.7))^{10}}{10!}\ e^{-2(0.6)}\frac{(2(0.6))^{9}}{9!}\ e^{-2(1.8)}\frac{(2(1.8))^{12}}{12!},$$

(Remove this question from sheet 3).

(6)
$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{pmatrix} \end{array}, \quad \pi = (0.5, 0.5)$$

$$P(X(3)=0) = P(X(3)=0 \mid X(0)=0)\ P(X(0)=0) + P(X(3)=0 \mid X(0)=1)\ P(X(0)=1)$$

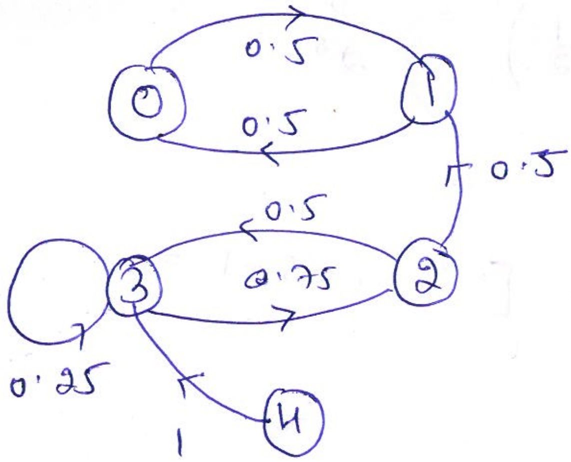
$$= P_{00}^{(3)}\left(\frac{1}{2}\right) + P_{10}^{(3)}\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\left(P_{00}^{(3)} + P_{10}^{(3)}\right) = \frac{1}{2}(0.2440 + 0.2520)$$

$$= \underline{0.2480}$$

$$\approx \underline{0.25}$$

(7)



$C(2) = \{2, 3\}$

period of state 1 is

$= g.c.d\{2, 4, 6, \ldots\} = 2$

period of state 2 is

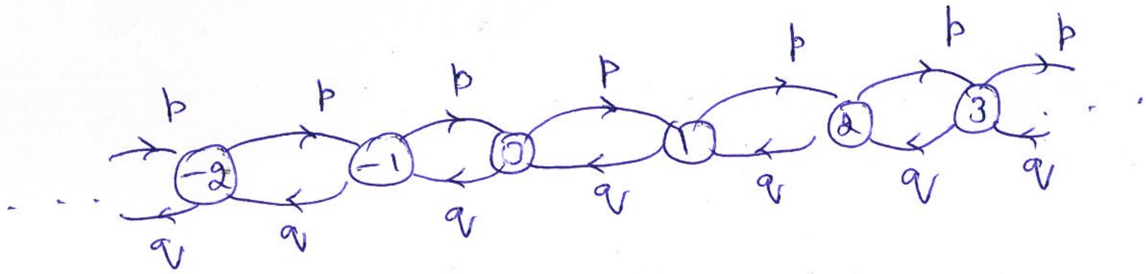
$= g.c.d\{2, 3, 4, 5, \ldots\} = 1$

state 0 and 2 are not communicating

$\therefore$ chain is reducible.

Hence, option (b) is NOT TRUE.

(8)



It can be observed that

$$P_{ij}^{(n)} = \begin{cases} {}^{n}C_{\frac{n+j-i}{2}} \; p^{\frac{n+j-i}{2}} \; q^{\frac{n-j+i}{2}} & \text{if } (n+j-i) \text{ even} \\ 0 & \text{otherwise} \end{cases}$$

$\therefore \quad P_{15}^{(6)} = {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 = \frac{6}{2^6}$

option (a) $\quad$ [$\because$ $6+5-1 = 10$ is even]

(9) from question (8),

$P_{14}^{(6)} = 0 \quad$ [$\because$ $6+4-1 = 9$ is odd]

option (c)