Unit 7 - week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-09-04, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each questions. In case of multiple answers partial marks will be awarded for every correct option chosen provided no incorrect option have been chosen. 0 marks are awarded for questions not attempted.

1) Let X_n be result of nth toss of a fair coin such that $X_n = 1$ if head occurs and 0 if tail occurs. Let $S_n = \sum_{i=1}^{n} X_i,\ n \geq 1$. Let Y_n be the remainder when S_n is divided by 3. Which of the following is Not TRUE? — *2 points*

- $\{Y_n, n = 1, 2, \ldots\}$ is an irreducible Markov chain.
- $\{Y_n, n = 1, 2, \ldots\}$ is an aperiodic Markov chain.
- $\{Y_n, n = 1, 2, \ldots\}$ has a unique stationary distribution.
- limiting distribution of $\{Y_n, n = 1, 2, \ldots\}$ does not exist.

No, the answer is incorrect.
Score: 0

Accepted Answers:
limiting distribution of $\{Y_n, n = 1, 2, \ldots\}$ does not exist.

2) Let X_n be result of nth toss of a fair dice. — *2 points*
Let $S_n = \sum_{i=1}^{n} X_i,\ n \geq 1$
Let Y_n be the unit place digit of S_n. Which of the following is Not TRUE?

- $\{Y_n, n = 1, 2, \ldots\}$ is an irreducible Markov chain.
- $\{Y_n, n = 1, 2, \ldots\}$ is an aperiodic Markov chain.
- $\{Y_n, n = 1, 2, \ldots\}$ has infinitely many stationary distribution.
- $\lim_{n\to\infty} P(Y_n = 1) = \frac{1}{10}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{Y_n, n = 1, 2, \ldots\}$ has infinitely many stationary distribution.

3) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and transition probability matrix — *2 points*

$$P = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & 0 & 0 & \frac{3}{8} \\ 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & \frac{5}{8} & 0 \end{pmatrix}$$

Which one of the following is a stationary distribution for the Markov chain?

- $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$
- $(\frac{1}{3}, 0, 0, \frac{2}{3})$
- $(0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
- $(\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

4) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $\{1, 2\}$. Which one of the following can't be TRUE? — *2 points*

- The chain has unique stationary distribution.
- The chain is irreducible.
- $P(X_n = 1 \mid X_0 = 1)$ does not converges as $n \to \infty$.
- Both the states are transient states.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Both the states are transient states.

5) Consider a Markov chain $\{X_n, n = 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and one-step transition probability matrix — *2 points*

$$P = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$ Which one of the following is FALSE?

- $\lim_{n\to\infty} P_{22}^{n} = 0$
- $\sum_{n=0}^{\infty} P_{22}^{n} < \infty$
- $\lim_{n\to\infty} P_{22}^{n} = 1$
- $P_{11} = \frac{1}{2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\lim_{n\to\infty} P_{22}^{n} = 1$

6) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{-100, -99, \ldots, 0, \ldots, 99, 100\}$ with transition probabilities — *2 points*
$P_{i,i+1} = P_{i,i-1} = 0.5\ \forall\ -99 \leq i \leq 99$
$P_{100,99} = P_{100,100} = P_{-100,-100} = P_{-100,-99} = 0.5$
Which one of the following is False?

- The chain is irreducible.
- All states are positive recurrent.
- All states are periodic with period 2.
- Stationary distribution exist.

No, the answer is incorrect.
Score: 0

Accepted Answers:
All states are periodic with period 2.

7) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1\}$ and one-step transition probability matrix — *2 points*

$$P = \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}$$

Which one of the following is TRUE?

- $\lim_{n\to\infty} P_{i,0}^{n}$ does not exist for any choice of $i \in S$.
- $\lim_{n\to\infty} P_{i,0}^{n}$ exists and is positive for every choice of $i \in S$.
- $\lim_{n\to\infty} P_{i,0}^{n}$ always exists but may be 0 for some choice of $i \in S$.
- $\lim_{n\to\infty} P_{i,1}^{n}$ always exists but may be 0 for some choice of $i \in S$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\lim_{n\to\infty} P_{i,0}^{n}$ exists and is positive for every choice of $i \in S$.

8) A total of m red and m white balls are distributed into two urns with m balls per urn. At each step, a ball is randomly selected from each urn and the two balls are interchanged. Let X_n be number of red balls in urn 1 after n^{th} step. Which of the following is true? — *2 points*

- $P_{ii} = \frac{2i(m-i)}{m^2}$
- $P_{ii} = \frac{(m-i)^2}{m^2}$
- $P_{ii} = \frac{i^2}{m^2}$
- $P_{ii} = \frac{i(m-i)}{m^2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P_{ii} = \frac{2i(m-i)}{m^2}$