Assignment 11 - Objective

The due date for submitting this assignment has passed. Due on 2020-04-15, 23:59 IST. As per our records you have not submitted this assignment.

1) For \( p > 0 \), define the function \( \| \cdot \|_p : \mathbb{R}^n \to [0, \infty) \) as
\[
\| x \|_p = \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \text{ for all } x = (x_1, \ldots, x_n) \in \mathbb{R}^n.
\]
Then
- \( \| \cdot \|_p \) is a norm for all \( p > 0 \).
- \( \| \cdot \|_p \) is a norm if and only if \( p > 1 \).
- \( \| \cdot \|_p \) is a norm if and only if \( p \geq 1 \).
- None of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
- \( \| \cdot \|_p \) is a norm if and only if \( p \geq 1 \).

2) For any \( x = (x_1, \ldots, x_n) \in \mathbb{R}^n \) and \( p \geq 1 \), define
\[
\| x \|_p = \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p},
\]
and \( \| x \|_{\infty} = \max \{ |x_i| : i = 1, \ldots, n \} \).
State whether True or False. If \( (x_k) \) is a sequence and \( x \in \mathbb{R}^n \) such that \( \| x_k - x \|_{\infty} \to 0 \), then \( \| x_k - x \|_p \to 0 \), for all \( p \geq 1 \).

- True
- False
State whether True or False.

3) Let \( f_n : \mathbb{R} \rightarrow \mathbb{R} \), \( n \in \mathbb{Z}^+ \) be functions such that \( f_n(x) \rightarrow 0 \), for all \( x \in \mathbb{R} \). Then the sequence \( (f_n) \) converges uniformly to the zero function.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers: True

4) State whether True or False.

There exists a function \( f : \mathbb{R} \rightarrow \mathbb{R} \) such that \( f \) is differentiable everywhere, \( f' \) is bounded and \( f \) is not uniformly continuous.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers: False

5) Let \( f_n : [a, b] \rightarrow \mathbb{R} \) be Riemann integrable functions, for all \( n \in \mathbb{Z}^+ \). Let \( f : [a, b] \rightarrow \mathbb{R} \) be Riemann integrable such that \( f_n \rightarrow f \) uniformly on \([a, b]\). Then \( \int_a^b f_n \rightarrow \int_a^b f \).

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers: True