(1) Find and classify the critical points of the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4,$$
for all $(x, y) \in \mathbb{R}^2$. \[4\]

(2) Find the points of local maximum and local minimum and saddle points of the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x, y) = xye^{-x^2-y^2},$$
for all $(x, y) \in \mathbb{R}^2$. \[4\]

(3) Find the maximum and minimum values of the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x, y) = xy^2,$$
for all $(x, y) \in \mathbb{R}^2$,
in the region $D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$. \[4\]

(4) Using Lagrange multipliers, find the maximum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x + y + z = 12$. \[4\]

(5) (a) For any $k \in \mathbb{R}$ and $\mathcal{R} = [a, b] \times [c, d]$, show that

$$\iint_{\mathcal{R}} k \, dA = k(b - a)(d - c).$$ \[1\]

(b) Use (a) to show that

$$0 \leq \iint_{\mathcal{R}} \sin \pi x \cos \pi y \, dA \leq \frac{1}{32},$$
where $\mathcal{R} = [0, 1/4] \times [1/4, 1/2]$. \[3\]