Assignment 7 - Objective

The due date for submitting this assignment has passed. **Due on 2020-03-18, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let \( f : \mathbb{R} \to \mathbb{R} \) be continuous and define
\[
F(x) = \int_0^x f(y) \, dy, \quad \text{for all } x \in \mathbb{R}.
\]
Then

- \( F'(x) = 3x^2 f(x^3) \), for all \( x \in \mathbb{R} \).
- \( F'(x) = f(x^3) \), for all \( x \in \mathbb{R} \).
- \( F'(x) \) may not exist for all \( x \in \mathbb{R} \).
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( F'(x) \) may not exist for all \( x \in \mathbb{R} \).

2) We say a function \( f : [a, b] \to \mathbb{R} \) has bounded variation if there exists \( K > 0 \) such that
\[
\sum_{i=0}^{n-1} |f(a_{i+1}) - f(a_i)| \leq K,
\]
for any \( a = a_0 < a_1 < \cdots < a_n = b \).
Which of the following statements is true for functions on \([a, b]\)?

- Every Riemann integrable function has bounded variation.
- Every function having bounded variation is Riemann integrable, but may not be bounded.
- Every function having bounded variation is bounded and is Riemann integrable.
None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
*Every function having bounded variation is bounded and is Riemann integrable.*

3) State whether True or False.

Let \( f : [a, b] \to \mathbb{R} \) be Riemann integrable and let \( s, t \in [a, b] \). Define \( F_s, F_t : [a, b] \to \mathbb{R} \) as

\[
F_s(x) = \int_s^x f(u) \, du \quad \text{and} \quad F_t(x) = \int_s^x f(u) \, du,
\]

for all \( x \in [a, b] \).

Then \( F_s - F_t \) is a constant function.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

4) State whether True or False.

Let \( f : [a, b] \to \mathbb{R} \). Let \( (P_k)_{k \geq 1} \) be a sequence of partitions of \( [a, b] \) such that

\[
\lim_{k \to \infty} (U(P_k, f) - L(P_k, f)) = 0.
\]

Then \( f \) is Riemann integrable.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

5) State whether True or False.

Let \( f : [a, b] \to \mathbb{R} \). For any \( c > 0 \) and \( t \in \mathbb{R} \), define \( f_{c, t} : [(a - t)/c, (b - t)/c] \to \mathbb{R} \) as

\[
f_{c, t}(x) = f(cx + t), \quad \text{for all } x \in [(a - t)/c, (b - t)/c].
\]

Then \( f \) is Riemann integrable on \( [a, b] \) if and only if \( f_{c, t} \) is Riemann integrable on \( [(a - t)/c, (b - t)/c], \) for all \( c > 0, \ t \in \mathbb{R} \).

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True