Assignment 5 - Objective

The due date for submitting this assignment has passed. Due on 2020-03-04, 23:59 IST. As per our records you have not submitted this assignment.

1) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Consider the following statements.

(a) $f$ is continuous.

(b) $f^{-1}(U)$ is open, for every open set $U \subseteq \mathbb{R}$.

(c) $f^{-1}(E)$ is closed, for every closed set $E \subseteq \mathbb{R}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(a), (b) and (c) are equivalent.

2) State whether True or False.

Let $f : \mathbb{R} \to \mathbb{R}$ such that $f^{-1}(K)$ is compact, for every compact set $K \subseteq \mathbb{R}$. Then $f$ is continuous.

True
False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

3)
State whether True or False.
Let \( f : (a, b) \to \mathbb{R} \) be uniformly continuous. Then there exists \( g : [a, b] \to \mathbb{R} \) such that \( g \) is uniformly continuous and \( g = f \) on \((a, b)\).

\[ \text{True} \]
\[ \text{False} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \text{True} \]

4) Choose the most appropriate single choice:

Let \( f : \mathbb{R} \to \mathbb{R} \) such that \( |f(x) - f(y)| \leq 2|x - y| \), for all \( x, y \in \mathbb{R} \). Consider the following statements.

(a) \( f \) is continuous.
(b) For any \( x_0 \in \mathbb{R} \), define the sequence \((y_n)_{n \geq 0}\) as \( y_0 = f(x_0) \), \( y_{n+1} = f(y_n) \), for all \( n \geq 0 \). Then \((y_n)\) is a convergent sequence.
(c) There exists \( x^* \in \mathbb{R} \) such that \( f(x^*) = x^* \).
(d) There exists unique \( x^* \in \mathbb{R} \) such that \( f(x^*) = x^* \).

\[ \text{(a), (b) and (c) are true, but (d) is not true.} \]
\[ \text{(a) and (b) are true, but (c) and (d) are not true.} \]
\[ \text{(a) is true, but (b), (c) and (d) are not true.} \]
\[ \text{(a), (b), (c), (d) are all true.} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \text{(a), (b), (c), (d) are all true.} \]

5) Let \( A \subseteq \mathbb{R} \) be a nonempty set such that every bounded sequence in \( A \) is convergent. Which \textbf{1 point} of the following is true?

\[ \text{A is a closed interval with nonzero length, not necessarily bounded.} \]
\[ \text{A is a closed and bounded interval with positive length.} \]
\[ \text{A is a singleton set.} \]
\[ \text{A is a finite set, with size possibly greater than 1.} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \text{A is a singleton set.} \]