(1) Consider the following homogeneous system of linear equations.

\[ x + 2y = 0, \quad ax + 8y + 3z = 0, \quad by + 5z = 0. \]

(a) Find a value of \( a \) which will make it necessary during Gaussian elimination to interchange rows in the coefficient matrix. \[2\]

(b) Suppose \( a \) does not have the value obtained in part (a). Find the values of \( b \) so that the system has a nontrivial solution. \[3\]

(c) Suppose \( a \) does not have the value obtained in part (a) and \( b = 100 \). Suppose further that the value of \( a \) is chosen so that the solution to the system is not unique. Find the general solution to the system. \[5\]

(2) Consider the following system of equations.

\[
\begin{align*}
x + y + z &= 2, \\
x + 3y + 3z &= 0, \\
x + 3y + 6z &= 3.
\end{align*}
\]

(a) Use Gaussian elimination to convert the coefficient matrix to REF. \[2\]

(b) Solve the system. \[3\]

(c) Let \( A \) denote the coefficient matrix given. The rowspace (columnspace) of \( A \) is the set of all linear combinations of the rows (columns) of \( A \). Find a basis of the rowspace and the columnspace of \( A \). \[5\]