NPTEL Basic Linear Algebra 2020
Assignment 2 - Subjective

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Deadline: Wednesday, February 12, 2020, 23:59 IST

(1) Give an example of the following types of linear systems, consisting of three equations in three unknowns, with justification. **[4]**

(a) inconsistent.

(b) consistent with no free variables.

(c) consistent with exactly one free variable.

(d) consistent with exactly two free variables.

(2) Which of the following are subspaces of $\mathbb{R}^3$? Justify. **[3]**

(a) The set of points $(x, y, z) \in \mathbb{R}^3$ such that $x^2 + y^2 = z$.

(b) The set of points $(x, y, z) \in \mathbb{R}^3$ such that $x + y + z = 0$, $x - y + z = 1$.

(c) The set of points $(x, y, z) \in \mathbb{R}^3$ such that $x = z$, $x = -z$.

(3) Show that $x = (1, 1, 0), y = (0, 1, 2), z = (3, 1, -4) \in \mathbb{R}^3$ are linearly dependent by finding $\alpha, \beta \in \mathbb{R}$ such that $\alpha x + \beta y + z = 0$. **[1]**

(4) Let $u = (\lambda, 1, 0), v = (1, \lambda, 1), w = (0, 1, \lambda) \in \mathbb{R}^3$. Find all values of $\lambda \in \mathbb{R}$ for which $\{u, v, w\}$ is linearly dependent. **[2]**

(5) Let $M_2(\mathbb{R})$ denote the set of all 2×2 matrices with real entries. Define

$$\mathcal{U} = \left\{ \begin{bmatrix} u & -u - x \\ 0 & x \end{bmatrix} : u, x \in \mathbb{R} \right\},$$

$$\mathcal{L} = \left\{ \begin{bmatrix} v & 0 \\ w & -v \end{bmatrix} : v, w \in \mathbb{R} \right\}.$$

Further, define $\mathcal{U} + \mathcal{L} = \{A + B : A \in \mathcal{U},\ B \in \mathcal{L}\}$.

(a) Show that $\mathcal{U}$ and $\mathcal{L}$ are subspaces of $M_2(\mathbb{R})$. **[2]**

(b) Find a basis for each of $\mathcal{U}$, $\mathcal{L}$, $\mathcal{U} + \mathcal{L}$ and $\mathcal{U} \cap \mathcal{L}$, with justification. **[8]**