Q.1. [5 marks]
Let \((X, S)\) be a measurable space. Let \(f, g : X \to \mathbb{R}^*\) be measurable functions and let \(\alpha \in \mathbb{R}\) with \(p > 1\) and \(m \in \mathbb{N}\). Prove the following.

(a) \(f + \alpha\) is a measurable function.

(b) Let \(\beta, \gamma \in \mathbb{R}^*\). Define

\[
    f^m(x) = \begin{cases} 
        (f(x))^m, & f(x) \in \mathbb{R} \\
        \beta, & f(x) = +\infty \\
        \gamma, & f(x) = -\infty 
    \end{cases}
\]

Then \(f^m\) is a measurable function.

(c) Let \(\beta, \gamma, \delta \in \mathbb{R}^*\). Define

\[
    (1/f)(x) = \begin{cases} 
        1/f(x), & f(x) \notin \{0, +\infty, -\infty\} \\
        \beta, & f(x) = 0 \\
        \gamma, & f(x) = +\infty \\
        \delta, & f(x) = -\infty 
    \end{cases}
\]

Then \(1/f\) is a measurable function.

Q.2. [2 marks]
Let \((X, S)\) be a measurable space and \(f : X \to \mathbb{R}^*\) be measurable. Show that \(|f|\) is also measurable. Give an example to show that the converse need not be true.
Q.3. [3 marks]
Let \((X, \mathcal{S})\) be a measurable space such that for every \(f : X \to \mathbb{R}\), we have
\[
f \text{ is measurable} \iff |f| \text{ is measurable}.
\]
Show that \(\mathcal{S} = \mathcal{P}(X)\).

Note. Check the Announcement page on the course portal for details on submitting the assignment solutions.