NPTEL Measure Theory, July 2018
Assignment 6

Deadline: Wednesday, September 12, 2018, 11.59 PM

Q.1. **[5 marks]**

Let $(X, \mathcal{S})$ be a measurable space. Let $f, g : X \to \mathbb{R}^*$ be measurable functions and let $\alpha \in \mathbb{R}$ with $p > 1$ and $m \in \mathbb{N}$. Prove the following.

(a) $f + \alpha$ is a measurable function.

(b) Let $\beta, \gamma \in \mathbb{R}^*$. Define

$$f^m(x) = \begin{cases} (f(x))^m, & f(x) \in \mathbb{R} \\ \beta, & f(x) = +\infty \\ \gamma, & f(x) = -\infty \end{cases}$$

Then f^m is a measurable function.

(c) Let $\beta, \gamma, \delta \in \mathbb{R}^*$. Define

$$(1/f)(x) = \begin{cases} 1/f(x), & f(x) \notin \{0, +\infty, -\infty\} \\ \beta, & f(x) = 0 \\ \gamma, & f(x) = +\infty \\ \delta, & f(x) = -\infty \end{cases}$$

Then $1/f$ is a measurable function.

Q.2. **[2 marks]**

Let $(X, \mathcal{S})$ be a measurable space and $f : X \to \mathbb{R}^*$ be measurable. Show that $|f|$ is also measurable. Give an example to show that the converse need not be true.

Q.3. **[3 marks]**

Let $(X, \mathcal{S})$ be a measurable space such that for every $f : X \to \mathbb{R}$, we have

$$f \text{ is measurable} \iff |f| \text{ is measurable.}$$

Show that $\mathcal{S} = \mathcal{P}(X)$.

Note. Check the Announcement page on the course portal for details on submitting the assignment solutions.