Measure Theory: Week 3 Assignment

Deadline: Wednesday, August 22, 2018, 11.59 PM

Q.1. **[4 marks]**

Let $F : \mathbb{R} \to \mathbb{R}$ be a monotonically increasing and right continuous function. Let $\tilde{\mathcal{I}}$ be the class of all left-open right-closed intervals in $\mathbb{R}$. Define $\mu_F : \tilde{\mathcal{I}} \to [0, \infty]$ by

$$\begin{aligned}
\mu_F(a, b] &= F(b) - F(a) \\
\mu_F(-\infty, b] &= \lim_{x\to\infty} (F(b) - F(-x)) \\
\mu_F(a, \infty) &= \lim_{x\to\infty} (F(x) - F(a)) \\
\mu_F(-\infty, \infty) &= \lim_{x\to\infty} (F(x) - F(-x))
\end{aligned}$$

Then prove that μ_F is countably additive.

Q.2. **[3 marks]**

Let $F(x) = [x] = \max\{n \in \mathbb{Z} \mid n \leq x\}$, for $x \in \mathbb{R}$, that is, $F(x)$ is the integral part of x. Describe the set function μ_F.

Hint. Observe and assume that F is monotonically increasing and right continuous.

Q.3. **[3 marks]**

Let F be a distribution function and $\alpha \in \mathbb{R}$. Show that $G = F + \alpha$ is also a distribution function and $\mu_G = \mu_F$.

Note. Check the Announcement page on the course portal for details on submitting the assignment solutions.