Q.1. [4 marks]

Let $F : \mathbb{R} \to \mathbb{R}$ be a monotonically increasing and right continuous function. Let $\mathcal{I}$ be the class of all left-open right-closed intervals in $\mathbb{R}$. Define $\mu_F : \mathcal{I} \to [0, \infty]$ by

$$
\mu_F(a, b] = F(b) - F(a)
$$
$$
\mu_F(-\infty, b] = \lim_{x \to \infty} (F(b) - F(-x))
$$
$$
\mu_F(a, \infty) = \lim_{x \to \infty} (F(x) - F(a))
$$
$$
\mu_F(-\infty, \infty) = \lim_{x \to \infty} (F(x) - F(-x))
$$

Then prove that $\mu_F$ is countably additive.

Q.2. [3 marks]

Let $F(x) = \lfloor x \rfloor = \max\{n \in \mathbb{Z} \mid n \leq x\}$, for $x \in \mathbb{R}$, that is, $F(x)$ is the integral part of $x$. Describe the set function $\mu_F$.

Hint. Observe and assume that $F$ is monotonically increasing and right continuous.

Q.3. [3 marks]

Let $F$ be a distribution function and $\alpha \in \mathbb{R}$. Show that $G = F + \alpha$ is also a distribution function and $\mu_G = \mu_F$.

Note. Check the Announcement page on the course portal for details on submitting the assignment solutions.