Decision Modeling (NPTEL Online Course)

Tutorial 4 (Queuing Theory - *Module 16 to Module 20)*

----

1. Which type of queuing system is likely to form in front of two counters in a small railway ticketing centre (one for enquiry and the other for ticket booking) with Poisson arrival and Exponential Service?
   i. One M/M/2
   ii. Two M/M/2
   iii. One M/M/1
   iv. Two M/M/1

2. Under M/D/1 model, average number of customers in the system is given by
   i. $\frac{\rho^2}{(1-\rho)}$
   ii. $\rho + \frac{\rho^2}{(1-\rho)}$
   iii. $\frac{1}{2}\frac{\rho^2}{(1-\rho)}$
   iv. $\boldsymbol{\rho + \frac{1}{2}\frac{\rho^2}{(1-\rho)}}$

3. In a small barber shop, only one customer can get hair cut while another customer can wait in a chair. Any other arriving customer has to wait outside as there is only one chair available. The customers arrive randomly at 6 per hour. The service is exponential and takes 6 minutes on the average. Find the probability that an arriving customer will have to wait outside.
   i. 36 %
   ii. 40 %
   iii. 60 %
   iv. 64 %

4. An overhead crane of ABC Ltd. moves jobs from one machine to another and must be used every time a machine requires loading or unloading. The demand for service is random. Data taken by recording the elapsed time between service calls followed an exponential distribution having a mean of a call every 24 minutes. In a similar manner, the actual service time of loading or unloading took an average of 8 minutes. If the machine time is valued at Rs. 8.50 per hour, how much does the downtime cost per day? [Assume 8 hour work per day]

i. 30
ii. 32
iii. 34
iv. 36

5. Arrivals to single bank counter are poisson distributed with a rate of 20 per hour. The average time for a customer to get service is 2 minutes and this time is exponentially distributed. What would be the average waiting time of a customer in the system?
   i. 2 minutes
   ii. 4 minutes
   iii. 6 minutes
   iv. 8 minutes

6. Consider the previous problem once again. Arrivals to single bank counter are poisson distributed with a rate of 20 per hour. The time for a customer to get service, however, is constant at 2 minutes. What would be the average waiting time of a customer in the system?
   i. 2 minutes
   ii. 4 minutes
   iii. 6 minutes
   iv. 8 minutes

7. A small railway ticket booking office has two counters – Counter 1 for enquiry and Counter 2 for ticket booking. Customer arrival is Poisson at 5 per hour to the enquiry and 10 per hour to the ticket booking counter. Exponentially distributed service time in each counter is 4 minutes per customer.

   Find by how much the average waiting time of a customer in the system reduces at Counter 1 (original enquiry counter) when the office decides to go for pooling of resources – i.e. an arriving customer will get enquiry or ticket booking facility at any of the counters.

   i. 0.333 minute
   ii. 0.667 minute
   iii. 2 minutes
   iv. 6.667 minutes

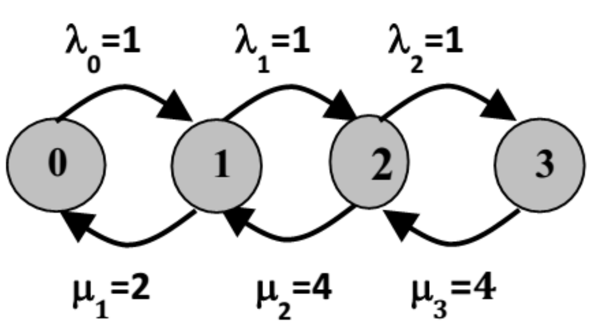
8. In a restaurant, customer arrival is Poisson at 10 per hour. In this restaurant, the customers do self-service. Exponentially distributed service time 3 minutes per customer. Find the average waiting time of a customer in the restaurant.
   i. 3 minutes
   ii. 6 minutes
   iii. 9 minutes
   iv. 12 minutes

9. Machines fail at 4 per hour and the cost of non-productive machine is Rs. 200 per hour. A repairman charges Rs. 100 per hour and repairs at 5 per hour. What will be the total queuing costs per hour? Assume M/M/1 queuing system.
    i. Rs. 300
    ii. Rs. 700
    iii. Rs. 900
    iv. Rs. 1000

10. In a tool crib, workers come to take tools at 4/hour on the average. Waiting for them costs Rs. 10/- per hour. The service time per worker is in the tool crib is 12 minutes. What will be total waiting cost of the workers per day if it is 8 hours a day? Assume M/M/1 queuing system.
    i. Rs. 40
    ii. Rs. 160
    iii. Rs. 320
    iv. Rs. 1280

11. In a single server tool-crib, mechanics come to take spares at 4/hour on the average. Waiting for them costs Rs. 8/- per hour. Average Waiting time for a mechanic in the system is W. What will be total waiting cost of the mechanics in a day for a 8 hour day?
    i. 8W
    ii. 48W
    iii. 64W
    iv. 256W

12. For the Rate diagram shown below, following relationship is true at Node 0:



i. $P_0 = P_1$
ii. $P_0 = 2P_1$
iii. $2P_0 = P_1$
iv. None of the above

13. Priority Discipline choices of a Queuing Model include: First Come First Served (FCFS), Service In Random Order (SIRO) and Last In First Served (LCFS) among others. Variances of waiting time for queuing models with FCFS, SIRO or LCFS have the following relationship:
i. $Var(W_{FCFS}) > Var(W_{SIRO}) > Var(W_{LCFS})$
ii. $Var(W_{FCFS}) = Var(W_{SIRO}) = Var(W_{LCFS})$
iii. $Var(W_{FCFS}) = Var(W_{LCFS}) < Var(W_{SIRO})$
iv. $Var(W_{FCFS}) < Var(W_{SIRO}) < Var(W_{LCFS})$

14. For obtaining optimum service capacity in a queuing system, we need to:
i. Minimize waiting costs only
ii. Minimize service cost only
iii. Minimize waiting and service cost together
iv. Operate at maximum possible service capacity

15. In a restaurant, two types of foods are available. Arrival rate of customers is 24 per hour Poisson and Average Service Time is 4 minutes exponential. The arriving customers can be served in three ways. The first way is to open two separate counters for the two types of foods. The second way is to pool the services, and offer both types of foods in both the counters. The third way is to open a large number (∞) of self-service counters for the customers. Which way will lead to the maximum busy period of the system?
i. The first way – separate counters
ii. The second way – pooled service
iii. The third way – self service counters
iv. Both the pooled service and the self-service

Explanations for selected problems

3) Average arrival rate $\lambda = 6$ per hour.

Average service time $1/\mu = 6$ minutes, and hence $\mu = 10$/hour

Hence, the utilization factor, $\rho = \lambda/\mu = 6/10 = 3/5$

So, the probability that an arrival will find the place free
= p(0) = 1 – ρ = 1 – (3/5) = 2/5

p (1) = ρ p(0) = 6/25

The probability that an arriving customer will have to wait outside = 1-[2/5 + 6/25] = 9/25 = 36%

4) Arrival rate = λ = $\frac{60}{24} = 2.5 \text{ per hour}$

Service rate = $\mu = \frac{60}{8} = 7.5 \text{ per hour}$

Average waiting time in the system = $\frac{1}{\mu - \lambda} = \frac{1}{7.5 - 2.5} = 0.20$

Average number of units in the system = $\frac{1}{\mu - \lambda} * \lambda = 0.5$

Number of hour per day is 8 hour

Cost per machine hour Rs. 8.50

Total downtime cost per day = $0.5 * 8 * 8.50 = \text{Rs.}34$

5) Arrival rate = $\lambda = 20 \text{ per hour}$

Service rate = $\mu =$ 2 minute per customer = 30 customers per hour

System Utilization factor = $\rho = \frac{\lambda}{\mu} = \frac{20}{30} = \frac{2}{3}$

Length of the system, L = $\frac{\rho}{1-\rho} = \frac{2/3}{1/3} = 2$

Average waiting time in system = $\frac{L}{\lambda} = \frac{2}{1/3} = 6 \text{ minutes}$

6) Since the service time is constant, the system is M/D/1

System Utilization factor = $\rho = \frac{\lambda}{\mu} = \frac{20}{30} = \frac{2}{3}$

Length of the system = $\rho + \frac{1}{2}\frac{\rho^2}{1-\rho} = \frac{8}{6}$

Average waiting time in system = $\frac{L}{\lambda} = \frac{8/6}{1/3} = 4$ minutes

7) We have two counters (i) Enquiry counter (ii) ticket booking counter

The arrival rate at the enquiry counter, $\lambda = 5$ per hour
The service rate at the enquiry counter, $\mu = 15$ per hour

The arrival rate at the ticket booking counter, $\lambda = 10$ per hour
The service rate at the ticket booking counter, $\mu = 15$ per hour

Case 1: Without pooling the resources

Enquiry counter

System Utilization factor, $\rho = \frac{5}{15} = \frac{1}{3}$

Average no. of customers in system, L = ρ/(1 – ρ) = (1/3)/(1 – 1/3) = 1/2

Average waiting time in system W = L/λ = (1/2)/5 = 1/10 Hour = 6 mins

Ticket booking counter

System utilization factor, ρ = λ/μ = 10/15 = 2/3

Average no. of customers in system, L = ρ/(1 – ρ) = (2/3)/(1 – 2/3) = 2

Average waiting time in system W = L/λ = 2/10 = 1/5 Hour = 12 mins

Case 2: With pooling the resources

Arrival rate of customers at the counters, λ = 5+10 = 15 per hour

Service rate at any one counter = 15 per hour

System utilization factor, ρ = λ/sμ = 15/(2*15) = ½

Using the M/M/2 queuing formula, we have,

$$P_0 = \left(\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \cdot \frac{1}{1-(\lambda/s\mu)} \right)^{-1}$$

$= [((15/15)^0/0!)+(15/15)^1/1!)+(15/15)^2/2!)*(1/(1–1/2))]^{-1} = [1+1+(1/2)*2]^{-1} =1/3$

$$L_q = \sum_{n=s}^{\infty}(n-s)P_n = ... = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2} P_0 \;\; = \frac{(15/15)^2 * (1/2)}{2!(1-1/2)^2} * (1/3) \;\; = \frac{1}{3}$$

Wq = Lq/λ =(1/3)/15 =1/45

W = Wq+(1/μ) =(1/45)+(1/15)=4/45 = 5.33 mins.

how much the average waiting time of a customer in the system reduces at Counter 1 (original enquiry counter) when the office decides to go for pooling of resources?

6 mins – 5.33 mins = **0.667 minute**

8) Arrival rate λ = 10/hour

Service rate = 3 mins per customer = 20/hour

System utilization factor, ρ = λ/μ = 10/20 = 1/2

So, $P_0 = e^{-\frac{\lambda}{\mu}} = e^{-\frac{1}{2}} = 0.606$ Hence, Busy Period = 1–0.606 = 0.394 = 39.4%

Average No. of customers in the restaurant $L = a = \frac{\lambda}{\mu} = \frac{1}{2}$

Average waiting time in the restaurant: $W = \frac{L}{\lambda} = \frac{1}{\mu} = \frac{1}{20}$ hour = **3 minutes**

9) Arrival rate λ = 4/hour

Service rate = 5/hour

System utilization factor, ρ = λ/μ = 4/5

Average number of machines in repair, L = ρ/(1 – ρ) = (4/5)/(1-4/5) = 4

Cost of non-productive machine is Rs. 200 per hour

(A) Total cost of Non productive machine is 200* 4 = Rs. 800

(B) Repairman's charge = Rs 100

Total Cost = (A) + (B) = **Rs. 900**

10) Arrival rate λ = 4/hour

Service rate = 5/hour

System utilization factor, $\rho = \lambda/\mu = 4/5$

Average number of workers in queue, $L = \rho/ (1 - \rho) = (4/5)/(1-4/5) = 4$

Waiting cost = Rs 10 per hour

Waiting cost for 4 workers per hour = Rs. 40

Waiting cost for 8 hours = **Rs. 320**

11) Arrival rate λ = 4/hour

Waiting cost = Rs 8 per hour

Average waiting time = w

Total waiting cost for 8 hours = 4*w*8*8 = **256w**