Decision Modeling (NPTEL Online Course)

Answer Key-Tutorial 3 (Queuing Theory - Module 11 to Module 15)

1. If the service rate increases and the arrival rate remains constant, then, in general
   i. Customer waiting time increases
   ii. Customer waiting time decreases
   iii. Service costs increase
   iv. Customer satisfaction decreases

2. Customer behavior in which the customer leaves a queue after joining is known as:
   i. Balking
   ii. Reneging
   iii. Jockeying
   iv. Alternating

3. Customer behavior in which the customer does not join a queue seeing its length is called:
   i. Balking
   ii. Reneging
   iii. Jockeying
   iv. Alternating

4. Which of the following relationships is not true
   i. \( W = W_q + 1/\mu \)
   ii. \( L = \lambda W \)
   iii. \( L = L_q + \lambda \)
   iv. \( L_q = \lambda W_q \)

5. As per Kendall notation, the letter ‘c’ indicates
   i. Probability distribution of the service time
   ii. Queue discipline
   iii. Number of servers in the system
   iv. Probability of distribution of inter arrival time

6. Which among the following is not a property of a Poisson process
   i. Stationarity
   ii. Orderliness
   iii. Independence
   iv. Linearity
7. Which priority discipline is usually applicable for choosing items that involves stacking in a pile and then taking out from them
   i. First Come First Serve
   ii. Last In First Out
   iii. Service In Random Order
   iv. None of the above

8. The condition \( p \left( x > t + \frac{\Delta t}{x} > \Delta t \right) = p(x > t) \) shows which of the following property of exponential distribution:
   i. Decreasing property
   ii. Memory-less property
   iii. Maximum entropy distribution
   iv. None of the above

9. In a bank counter, last arrival was at 11 AM. Arrival rate \( \lambda = 2/\text{hour} \). No. of arrivals is Poisson and Inter-arrival time is exponential. What is probability that the next arrival is after 1 PM?
   i. \( e^{-2} \)
   ii. \( 1-e^{-2} \)
   iii. \( e^{-4} \)
   iv. \( 1-e^{-4} \)

10. In a bank counter, last arrival was at 11 AM. Arrival rate \( \lambda = 2/\text{hour} \). No. of arrivals is Poisson and Inter-arrival time is exponential. Nobody has arrived till 12 noon. What is probability that the next arrival is before 1 PM?
    i. \( e^{-2} \)
    ii. \( 1-e^{-2} \)
    iii. \( e^{-4} \)
    iv. \( 1-e^{-4} \)

11. Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. What is the average waiting time of a customer in the system?
    i. 225 seconds
    ii. 200 seconds
    iii. 100 seconds
    iv. 125 seconds
12. Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. What is the average waiting time of a customer in the queue?
   i.  225 seconds
   ii. 200 seconds
   iii. 100 seconds
   iv.  **125 seconds**

13. A bank plans to open a single server drive-in banking facility at a particular centre. It is estimated that 28 customers will arrive each hour on an average. If, on an average, it requires 2 minutes to process a customer’s transaction. In this system what is the average waiting time of a customer in the system?
   i.  34 minutes
   ii. 32 minutes
   iii. **30 minutes**
   iv.  28 minutes

14. A bank plans to open a single server drive-in banking facility at a particular centre. It is estimated that 28 customers will arrive each hour on an average. If, on an average, it requires 2 minutes to process a customer’s transaction. How many cars should then be accommodated in the driveway on the average including the car in the service?
   i.  7
   ii. **14**
   iii. 21
   iv.  28

15. An AC serviceman finds that the time spent on his job has an exponential distribution with a mean of 18 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 3 per hour. The serviceman works for 8 hours a day. What is the serviceman’s expected idle time each day?
   i.  18 minutes
   ii. 30 minutes
   iii. **48 minutes**
   iv.  1 hour
Explanations

Questions 1-8 are direct and the answers are available in the lecture material. We provide hints and explanations for questions 9-15 as follows

9) Required probability = \( p(x > t) = e^{-\lambda t} = e^{-2t} = e^{-4} \)

10) Required probability = \( p(x < t) = 1 - e^{-\lambda t} = 1 - e^{-2t} = 1 - e^{-2} \)

11) Arrival rate, \( \lambda = 20 \) per hour

Service rate, \( \mu = 36 \) per hour

Average waiting time of a customer in the system = \( \frac{1}{\mu - \lambda} = \frac{60*60}{36} = 225 \) seconds

12) Average waiting time of a customer in the queue = \( \frac{\lambda}{\mu(\mu - \lambda)} = \frac{20*60*60}{36*16} = 125 \) seconds

13) Arrival rate, \( \lambda = 28 \) per hour

Service rate, \( \mu = 30 \) per hour

Average waiting time of a customer in the system = \( \frac{1}{\mu - \lambda} = \frac{60}{2} = 30 \) minutes

14) Average number of customers in the queue = \( \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{28^2}{30*(30-28)} = 13.067 \approx 14 \)

15) Arrival rate, \( \lambda = 3 \) per hour

Service rate, \( \mu = \frac{60}{18} = 3.33 \) per hour

Utilization, \( \rho = \frac{\lambda}{\mu} = \frac{3}{3.33} = 0.90 \)

Idle time = \((1 - \rho)*8*60 = (1-0.9)*480 = 48 \) minutes