Question #1
(a) Replace X by Y

(a) The expected profit of the insurance company is \( qX - pX - pcX = [q - (1 + c)p]X \).
(You might have thought that the administration cost was \( cX \). But remember, this cost is incurred only if a claim is made, that is, with probability \( p \). Therefore the expected cost is \( pcX \).) With several perfectly competitive insurance companies, in equilibrium the expected profit must be zero, therefore \( q = (1 + c)p \).

(b) Call the scenario in which you suffer the loss scenario 1, and the one where your wealth stays intact, scenario 2. If you take out coverage \( X \), your expected utility is

\[
EU = \frac{1}{1 - r} \left[ p \left\{ \frac{1}{2} W_0 + (1 - q)X \right\}^{1-r} + (1 - p) \left\{ W_0 - qX \right\}^{1-r} \right]
\]

when \( r \neq 1 \) (and the log case if \( r = 1 \)).

(c) To choose \( X \) to maximize this, the FONC is

\[
\frac{dEU}{dX} = p \left\{ \frac{1}{2} W_0 + (1 - q)X \right\}^{r-1} (1 - q) + (1 - p) \left\{ W_0 - qX \right\}^{-r} (-q) = 0
\]

(The derivative \( dEU/dX \) has the same functional form for all cases of \( r \), so you actually don’t need to do the log case separately.)

Then

\[
\frac{\frac{1}{2} W_0 + (1 - q) X}{W_0 - q X} = \left( \frac{(1 - p) q}{p (1 - q)} \right)^{1/r}
\]

Second-order conditions are OK because the wealth in each scenario is a linear function of \( X \), the utility-of-consequences function in each scenario is a concave function of the wealth in that scenario, and expected utility is a positive linear combination of the utilities in the various scenarios.

(d) When \( p = 0.10 \), for two values of the administrative cost factor \( c = 0.1 \) and \( 0.2 \), we have \( q = 0.11 \) and \( 0.12 \) respectively. Then, for the three values of the risk aversion coefficient \( r \) given, we have the following table for the resulting values of the coverage ratio \( X/(\frac{1}{2} W_0) \):

<table>
<thead>
<tr>
<th>Cost factor c</th>
<th>Relative risk aversion ( r )</th>
<th>0.25</th>
<th>1.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
<td>0.306</td>
<td>0.798</td>
<td>0.979</td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>-0.126</td>
<td>0.643</td>
<td>0.962</td>
</tr>
</tbody>
</table>

The formula yields negative coverage for the case \( r = 0.25 \), \( c = .2 \) because with such low risk aversion and the high load factor, you would not wish to purchase any insurance. The true optimum is an extreme of \( X = 0 \).
(b)

A) The return on the risk free asset is given as 8%. The standard deviation of that return is 0 by definition, since the asset is risk free.

B) Expected return is given by:

\[ R = X_M R_M + X_F R_F + (.5)(.16) + (.5)(.08) = 12\% \]

The correlation coefficient between the risk free asset and the market portfolio is 0 by definition. In addition, since the standard deviation of the return on the risk free asset is also 0, the standard deviation of the portfolio is:

\[ \sigma = X_M \sigma_M = (.5)(.20) = .10 \]

C) If 100% of the investor's wealth is put into the market portfolio, the risk and return will equal that of the market portfolio. In this case the return will be 16%, and the standard deviation of return will be .20.

D) The standard deviation of return will be equal to:

\[ \sigma = X_M \sigma_M = (1.25)(.20) = .25 \]

Return will be equal to:

\[ R = X_M R_M + (1-X_M)R_F = 1.25(.16) + (-.25)(.08) = 18\% \]

This result can also be obtained using:

\[ R = R_F + \frac{R_M - R_F}{\sigma_M} \sigma = .08 + \frac{16 - .08}{.20} = 18\% \]

Question #2

Question to be done in an EXCEL file

Question #3

(a)

Each period is of length \( h = 0.25 \). Using the first two formulas on page 332 of McDonald (2006):

\[ u = \exp[-0.01\times0.25 + 0.3\times \sqrt{0.25}] = \exp(0.1475) = 1.158933, \]
\[ d = \exp[-0.01\times0.25 - 0.3\times \sqrt{0.25}] = \exp(-0.1525) = 0.858559. \]

Using formula (10.13), the risk-neutral probability of an up move is

\[ p^* = \frac{e^{-0.01\times0.25} - 0.858559}{1.158933 - 0.858559} = 0.4626. \]

The risk-neutral probability of a down move is thus 0.5374. The 3-period binomial tree for the exchange rate is shown below. The numbers within parentheses are the payoffs of the put option if exercised.
The payoffs of the put at maturity (at time 3h) are
$P_{uuu} = 0$, $P_{uud} = 0$, $P_{udd} = 0.3384$ and $P_{ddd} = 0.6550$.

Now we calculate values of the put at time 2h for various states of the exchange rate.

If the put is European, then
$P_{uu} = 0$,
$P_{ud} = e^{-0.02}[0.4626P_{uud} + 0.5374P_{udd}] = 0.1783$,
$P_{dd} = e^{-0.02}[0.4626P_{udd} + 0.5374P_{ddd}] = 0.4985$.

But since the option is American, we should compare $P_{uu}$, $P_{ud}$ and $P_{dd}$ with the values of the option if it is exercised at time 2h, which are 0, 0.1371 and 0.5059, respectively.

Since $0.4985 < 0.5059$, it is optimal to exercise the option at time 2h if the exchange rate has gone down two times before. Thus the values of the option at time 2h are $P_{uu} = 0$, $P_{ud} = 0.1783$ and $P_{dd} = 0.5059$.

Now we calculate values of the put at time h for various states of the exchange rate.

If the put is European, then
$P_{u} = e^{-0.02}[0.4626P_{uu} + 0.5374P_{ud}] = 0.0939$,
$P_{d} = e^{-0.02}[0.4626P_{ud} + 0.5374P_{dd}] = 0.3474$.

But since the option is American, we should compare $P_{u}$ and $P_{d}$ with the values of the option if it is exercised at time h, which are 0 and 0.3323, respectively. Since $0.3474 > 0.3323$, it is not optimal to exercise the option at time h. Thus the values of the option at time h are $P_{u} = 0.0939$ and $P_{d} = 0.3474$.

Finally, discount and average $P_{u}$ and $P_{d}$ to get the time-0 price,

$P = e^{-0.02}[0.4626P_{u} + 0.5374P_{d}] = 0.2256$.

Since it is greater than 0.13, it is not optimal to exercise the option at time 0 and hence the price of the put is 0.2256.
Question # 4

(a)

a) Interest saving for company X = [MIBOR + 0.2%](case 1) − [MIBOR + 8.25% - 8.15%](case 2) = 0.10%, i.e., net saving = 0.001 * 7½ * 3½ *10000000 = Rs. 262500.

b) Interest saving for company Y = [9.05%](case 1) − [MIBOR + 0.8% + 8.15 - MIBOR](case 2) = 0.10%, i.e., net saving = 0.001 * 7½ * 3½ *10000000 = Rs. 262500.

c) Interest saving for company X and Y is = 0.20% (0.1%*2) and net saving = 0.002* 7½ 3½ *10000000 = Rs. 525000.

d) Interest saving for company X = [MIBOR + 0.2%](case 1) − [MIBOR + 8.25% - 8.10%](case2) = 0.05%, hence the net saving = 0.0005 * 7½ * 3½ *10000000 = Rs. 131250.

e) Interest saving for company Y = [9.05%](case 1) − [MIBOR + 0.80% + 8.20% - MIBOR](case2) = 0.05%, hence the net saving = 0.0005 * 7½ * 3½ *10000000 = Rs. 131250.

f) Interest earned by the financial institution = [MIBOR + 8.20% - MIBOR − 8.10%] = 0.10%, hence profit = 0.001 * 5½ * 7½ 100000000 = 262500.

(b)

According to put-call parity, the price of the put should be
Call price + PV of Strike price − Prepaid forward stock price
= 2.00 + 50e^{ −.5(0.03)} − 49.70e^{ −.5(0.02)} = 2.05.

Since the put is priced at 2.35, we should be able to sell the put at $2.35 and by a synthetic put for $2.05. The arbitrage profit will be $.30.

The synthetic put can be bought by shorting e^{ −.5(0.02)} shares of stock and receiving $49.21, and investing 50e^{ −.5(0.03)} = 49.26 (lending) at the risk-free rate for 6 months and buying the call for 2.00, for a net cost of 49.26 + 2.00 − 49.21 = 2.05.

Answer: B
(b) We are provided with the following information:

<table>
<thead>
<tr>
<th>Company</th>
<th>JY</th>
<th>FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>6.0%</td>
<td>11.5%</td>
</tr>
<tr>
<td>Company B</td>
<td>8.2%</td>
<td>13.3%</td>
</tr>
</tbody>
</table>

Independent actions by A and B:

Joint action by A and B through an intermediary the FI (one scenario), we can have different values to illustrate this:

A faces risk due to JY and FF; ii) B faces risk due to JY and FF; iii) **FI makes a profit of 0.1% on JY**

Joint action by A and B through an intermediary the FI (second scenario) we can have different values to illustrate this:

A faces risk due to JY and FF; ii) B faces risk due to JY and FF; iii) **FI makes a profit of 0.1% on FF**