Answer#1

1. 
   a. 3-Period Moving Average: $F_{3\text{MA}} = (A_{\text{March}} + A_{\text{April}} + A_{\text{May}})/3 = (38 + 39 + 43)/3 = 40$
   5-Period Moving Average: $F_{5\text{MA}} = (A_{\text{January}} + A_{\text{February}} + A_{\text{March}} + A_{\text{April}} + A_{\text{May}})/5$
      $= (32 + 41 + 38 + 39 + 43)/5 = 38.6$
   b. Naïve: $F_{\text{Naïve}} = A_{\text{May}} = 43$
   c. 3-Period Moving Average: $F_{3\text{HP}} = (A_{\text{April}} + A_{\text{May}} + A_{\text{June}})/3 = (39 + 43 + 41)/3 = 41$
   5-Period Moving Average: $F_{5\text{HP}} = (A_{\text{February}} + A_{\text{March}} + A_{\text{April}} + A_{\text{May}} + A_{\text{June}})/5$
      $= (41 + 38 + 39 + 43 + 41)/5 = 40.4$
   Naïve: $F_{\text{Naïve}} = A_{\text{June}} = 41$

d. 

<table>
<thead>
<tr>
<th>Month</th>
<th>Actual</th>
<th>3-Period</th>
<th>Absolute</th>
<th>5-Period</th>
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<td>1</td>
<td>38.6</td>
<td>2.4</td>
<td>43</td>
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MAD(3-period moving average) = $\frac{\sum |Actual - Forecast|}{n} = (2 + 3.67 + 1)/3 = 2.22$
MAD(5-period moving average) = $\frac{\sum |Actual - Forecast|}{n} = 2.4/1 = 2.4$
MAD(Naïve) = $\frac{\sum |Actual - Forecast|}{n} = (9 + 3 + 1 + 4 + 2)/5 = 3.8$

The 3-period moving average provides the best historical fit using the MAD criterion and would be better to use.

e. 

<table>
<thead>
<tr>
<th>Month</th>
<th>Actual</th>
<th>3-Period</th>
<th>Squared</th>
<th>5-Period</th>
<th>Squared</th>
<th>Naïve</th>
<th>Squared</th>
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<td>Error</td>
<td>Moving</td>
<td>Error</td>
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<tr>
<td>January</td>
<td>32</td>
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<td>38</td>
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<tr>
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<td>43</td>
<td>39.33</td>
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<tr>
<td>June</td>
<td>41</td>
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<td>1</td>
<td>38.6</td>
<td>5.76</td>
<td>43</td>
<td>4</td>
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</table>

MSE(3-period moving average) = $\frac{\sum (Actual - Forecast)^2}{n-1} = (4 + 13.47 + 1)/2 = 9.24$
MSE(5-period moving average) = $\frac{\sum (Actual - Forecast)^2}{n-1}$. Not possible to compute since there are not enough observations (i.e., n = 1).
MSE(Naïve) = $\frac{\sum (Actual - Forecast)^2}{n-1} = (81 + 9 + 1 + 16 + 4)/4 = 111/4 = 27.75$

The 3-period moving average provides the best historical fit using the MSE criterion.
2.

Forecasts using $\alpha = 0.1$:

<table>
<thead>
<tr>
<th>Week</th>
<th>Demand</th>
<th>Exponential Smoothing</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>330</td>
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<tr>
<td>2</td>
<td>350</td>
<td>330</td>
<td>20</td>
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<tr>
<td>3</td>
<td>320</td>
<td>332</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>370</td>
<td>330.8</td>
<td>39.2</td>
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<tr>
<td>5</td>
<td>368</td>
<td>334.72</td>
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<tr>
<td>6</td>
<td>343</td>
<td>338.048</td>
<td>4.962</td>
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</table>

MAD: 21.89

Forecasts using $\alpha = 0.7$:

<table>
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<tr>
<th>Week</th>
<th>Demand</th>
<th>Exponential Smoothing</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>330</td>
<td>330</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>350</td>
<td>330</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>320</td>
<td>344</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>370</td>
<td>327.2</td>
<td>42.8</td>
</tr>
<tr>
<td>5</td>
<td>368</td>
<td>357.16</td>
<td>10.84</td>
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<tr>
<td>6</td>
<td>343</td>
<td>364.748</td>
<td>21.748</td>
</tr>
</tbody>
</table>

MAD: 23.68

Using $\alpha = 0.1$ provides a better historical fit based on the MAD criterion.

3.

Given: $T_4 = 20$, $A_5 = 90$, $S_4 = 83$

**Step 1:**
Smoothing the level of the series:

$$S_5 = \alpha A_5 + (1 - \alpha)(S_4 + T_4) = 0.20(90) + 0.80(85 + 20) = 102$$

**Step 2:**
Smoothing the trend:

$$T_5 = \beta(S_5 - S_4) + (1 - \beta)T_4 = 0.10(102 - 85) + 0.90(20) = 19.7$$

**Step 3:**
Forecast Including Trend

$$FIT_5 = S_5 + T_5 = 102 + 19.7 = 121.7$$
The prices are not arbitrage-free. To show that Mary’s portfolio yields arbitrage profit,

\[
\begin{array}{|c|c|c|c|c|}
\hline
& \text{Time 0} & \text{Time } T \\
\hline
\text{Buy 1 call} & -11 & 0 & \sigma^T & \sigma^T \\
\text{Strike 40} & & & & \\
\text{Sell 3 calls} & +18 & 0 & 0 & -3(S_T-50) \\
\text{Strike 50} & & & & -3(S_T-50) \\
\text{Lend $1} & -1 & \sigma^T & \sigma^T & \sigma^T \\
\text{Buy 2 calls} & -6 & 0 & 0 & 2(S_T-55) \\
\text{strike 55} & & & & \\
\text{Total} & 0 & \sigma^T > 0 & \sigma^T + S_T-40 > 0 & \sigma^T + 2(55-S_T) > 0 \\
\hline
\end{array}
\]

Peter’s portfolio makes arbitrage profit, because:

\[
\begin{array}{|c|c|c|}
\hline
& \text{Time-0 cash flow} & \text{Time-T cash flow} \\
\hline
\text{Buy 2 calls} & 2(-3 + 11) = 16 & 2(S_T-55) \\
\text{& sells 2 puts} & & \\
\text{Strike 55} & & \\
\text{Buy 1 call} & -11 + 3 = -8 & S_T-40 \\
\text{& sell 1 put} & & \\
\text{Strike 40} & & \\
\text{Lend $2} & -2 & 2\sigma^T \\
\text{Sell 3 calls} & 3(6-8) = -6 & 3(50-S_T) \\
\text{& buy 3 puts} & & \\
\text{Strike 50} & & \\
\text{Total} & 0 & 2\sigma^T \\
\hline
\end{array}
\]

b) The payoff at the contract maturity date is 
\[
\pi \times (1 - y\%) \times \max\{S(T)/S(0), (1 + g\%)^T\} \\
= \pi \times (1 - y\%) \times \max\{S(1)/S(0), (1 + g\%)^T\} \text{ because } T = 1 \\
= \frac{\pi}{S(0)} \times (1 - y\%) \times \max\{S(1), (1 + g\%)^T\} \text{ because } g = 3 \text{ & } S(0) = 103 \\
= \frac{\pi}{100} \times (1 - y\%) \times (S(1) + \max\{0, 103 - S(1)\}) \\
\]

Now, \(\max\{0, 103 - S(1)\}\) is the payoff of a one-year European put option, with strike price $103, on the stock index; the time-0 price of this option is given to be is $15.21. Dividends are incorporated in the stock index (i.e., \(\delta = 0\)); therefore, \(S(0)\) is the time-0 price for a time-1 payoff of amount \(S(1)\). Because of the no-arbitrage principle, the time-0 price of the contract must be 
\[
\frac{\pi}{100} \times (1 - y\%) \times (S(0) + 15.21) \\
= \frac{\pi}{100} \times (1 - y\%) \times 115.21. \\
\]

Therefore, the “break-even” equation is 
\[
\pi = \frac{\pi}{100} \times (1 - y\%) \times 115.21, \\
or \]
\[
y\% = 100 \times (1 - 1/1.1521) = 13.202\% \\
\]
Answer #3:

a) First, we construct the two-period binomial tree for the stock price.

\[
\begin{array}{ccc}
\text{Year 0} & \text{Year 1} & \text{Year 2} \\
20 & 25.680 & 32.9731 \\
 & 22.1028 & \\
 & 17.214 & 14.8161
\end{array}
\]

The calculations for the stock prices at various nodes are as follows:

\[
S_u = 20 \times 1.2840 = 25.680 \\
S_d = 20 \times 0.8607 = 17.214 \\
S_{uu} = 25.68 \times 1.2840 = 32.9731 \\
S_{ud} = S_{du} = 17.214 \times 1.2840 = 22.1028 \\
S_{dd} = 17.214 \times 0.8607 = 14.8161
\]

The risk-neutral probability for the stock price to go up is

\[
p^* = \frac{e^{rT} - d}{u - d} = \frac{e^{0.05} - 0.8607}{1.2840 - 0.8607} = 0.4502.
\]

Thus, the risk-neutral probability for the stock price to go down is 0.5498.

If the option is exercised at time 2, the value of the call would be

\[
C_{uu} = (32.9731 - 22)_+ = 10.9731 \\
C_{ud} = (22.1028 - 22)_+ = 0.1028 \\
C_{dd} = (14.8161 - 22)_+ = 0
\]

If the option is European, then
\[
C_u = e^{-0.05}[0.4502C_{uu} + 0.5498C_{ud}] = 4.7530
\]
\[
C_d = e^{-0.05}[0.4502C_{ud} + 0.5498C_{dd}] = 0.0440
\]

But since the option is American, we should compare \( C_u \) and \( C_d \) with the value of the option if it is exercised at time 1, which is 3.68 and 0, respectively. Since 3.68 < 4.7530 and 0 < 0.0440, it is not optimal to exercise the option at time 1 whether the stock is in the up or down state. Thus the value of the option at time 1 is either 4.7530 or 0.0440.

Finally, the value of the call is

\[
C = e^{-0.05}[0.4502(4.7530) + 0.5498(0.0440)] = 2.0585.
\]
\[ C(S, K, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2) \]

with
\[ d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

Because \( S = 20, K = 25, \sigma = 0.24, r = 0.05, T = 3/12 = 0.25 \), and \( \delta = 0.03 \), we have
\[ d_1 = \frac{\ln(20/25) + (0.05 - 0.03 + \frac{1}{2} \cdot 0.24^2)0.25}{0.24 \sqrt{0.25}} = -1.75786 \]

and
\[ d_2 = -1.75786 - 0.24 \sqrt{0.25} = -1.87786 \]

Using the Cumulative Normal Distribution Calculator, we obtain \( N(-1.75786) = 0.03939 \) and \( N(-1.87786) = 0.03020 \).

Hence, formula (12.1) becomes
\[ C = 20e^{-0.03 \times 0.25} (0.03939) - 25e^{-0.05 \times 0.25} (0.03020) = 0.036292362 \]

Cost of the block of 100 options = 100 x 0.0363 = \$3.63.

**OBJECTIVE**

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<tr>
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<td>8</td>
<td>D</td>
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<td>9</td>
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<td>10</td>
<td>The average return on Gold is much less than on the stock market</td>
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<td>11</td>
<td>14% and 167.25</td>
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<td>The optimal amount to invest in gold would drop</td>
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