1. (5 marks) Let \( p, q \in [0, 1] \). Prove Pinsker’s inequality, i.e., \( d(p, q) \geq 2(p - q)^2 \).

**Hint:** Consider the function \( g(x) = d(p, p + x) - 2x^2 \) over the \([-p, 1 - p]\) interval. By taking derivatives, show that \( g \geq 0 \).