Assignment-4

1. The discount process is defined by $D(t) = e^{-\int_0^t R(s)\,ds}$. Then $\frac{d}{dt}D(t) =$ ?
   
   (a) $-e^{-\int_0^t R(s)\,ds}$
   (b) $-e^{-\int_0^1 R(s)\,ds}$
   (c) $-R(t)e^{-\int_0^t R(s)\,ds}$
   (d) None of these

2. Suppose $W(t)$ is a Brownian motion ($t \in [0, T]$). Then for $t \in [0, T]$ $Y = \frac{W(T) - W(t)}{\sqrt{T - t}}$ follows:
   
   (a) $N(0, \sqrt{T - t})$
   (b) $N(t, \sqrt{T - t})$
   (c) $N(0, 1)$
   (d) $N(0, T - t)$

3. Every martingale can be represented as an Ito-Integral.
   
   (a) True
   (b) False

4. Suppose at time $t = 0$ you invest $S_0$ amount in risk asset. At time $t = 1$ you possibly receive either amount $S_1 = uS_0$ or amount $S_1 = dS_0$ where $u > d$. Suppose $r$ is fixed interest rate, then what is risk neutral probability $\tilde{q}$ of obtaining amount $dS_0$?
   
   (a) $(u - 1 - r)/(u - d)$
   (b) $(u + 1 - r)/(u + d)$
   (c) $(u + 1 + r)/(u + d)$
   (d) $(1 + r - d)/(u - d)$

5. In a market, stock prices never follows geometric Brownian motion.
   
   (a) True
   (b) False
6. Suppose at time \( t = 0 \) you invest \( S_0 \) amount in risk asset. At time \( t = 1 \) you possibly receive either amount \( S_1 = uS_0 \) or amount \( S_1 = dS_0 \) where \( u > d \). Suppose \( r \) is fixed interest rate, then no arbitrage condition is?

(a) \( 0 < d < 1 + r < u \)
(b) \( 0 < 1 + r < d < u \)
(c) \( 0 < d < u < 1 + r \)
(d) None of these

7. Suppose at time \( t = 0 \) you invest \( S_0 \) amount in a market. At time \( t = 1 \) you receive amount \( S_1 \). If \( K \) is strike price then value of option at \( t = 1 \) is given by

(a) \( \min\{S_1 - K, 0\} \)
(b) \( \max\{S_1 - K, 0\} \)
(c) \( \min\{S_0 - K, 0\} \)
(d) \( \max\{S_0 - K, 0\} \)

8. Suppose at time \( t = 0 \) you invest \( S_0 \) amount in risk asset. At time \( t = 1 \) you possibly receive either amount \( S_1 = uS_0 \) or amount \( S_1 = dS_0 \) where \( u > d \). Suppose \( r \) is fixed interest rate, then what is risk neutral probability \( \tilde{p} \) of obtaining amount \( uS_0 \)?

(a) \( \frac{1 - r - d}{u - d} \)
(b) \( \frac{1 - r - d}{u + d} \)
(c) \( \frac{1 + r + d}{u + d} \)
(d) \( \frac{1 + r - d}{u - d} \)

9. Under risk neutral probabilities, the discounted stock price is a Martingale.

(a) True
(b) False

10. The measure of ups and downs in the stock prices is stock volatility.

(a) True
(b) False

Answers

1. c
2. c
3. a
4. b
5. a
6. a
7. b
8. d
9. a
10. a