Assignment-1

Solutions:

1. (b). By the definition of Ideal bank.

2. (c). If a person is depositing A rs to the bank with simple interest rate r% then total amount after n years will be A(1+nr). After putting the values given, the amount will be 160.

3. (a). If a person is depositing A rs to the bank with compound interest rate r% then total amount after n years will be \( A(1 + r)^n \). After putting the values given, the amount will be 121.

4. (d). The proof is done in the lecture 2.

5. (a). For \( f(x) = |x|, \lambda \in [0,1] \ x \in \mathbb{R} \), apply the definition of convexity,

\[
\begin{align*}
 f(\lambda x + (1 - \lambda)y) &= |\lambda x + (1 - \lambda)y| \\
 &\leq |\lambda x| + |(1 - \lambda)y| \\
 &= \lambda|x| + (1 - \lambda)|y| \\
 &= \lambda f(x) + (1 - \lambda)f(y)
\end{align*}
\]

Hence \( f \) is convex function.

Now for \( g(x) \) and \( h(x) \) we can see \( g''(x) = 2 \geq 0 \) and \( h''(x) = 0 \geq 0 \) \( \forall x \in \mathbb{R} \). So second order condition for convexity will imply that \( g \) and \( h \) are convex function.

6. (d). For \( S_1 \), take \((-1,1) \in S_1 \) and \((1,-1) \in S_1 \) if we join these two points by a line then 0 will be mid point of that line but \( 0 \notin S_1 \). So \( S_1 \) is not a convex set.

For \( S_2 \), take \((1,1) \in S_2 \) and \((-1,-1) \in S_2 \) if we join these two points by a line then the whole line is not in the set. So \( S_2 \) is not convex.

Now \( S_3 \) is nothing but \( \mathbb{R}_+^2 \) i.e. nonnegative orthant. So if we take any two points in this set whole line joining these two points will be in this set.

7. (a). The proof is done in the lecture 3.

8. (b). By the definition of forward rate.
9. (b). Since the constraint set $x^2 = 1$, $x \in \mathbb{R}$ is same as $\{-1, 1\} \subset \mathbb{R}$ which is not a convex set, the given problem is not a convex optimization problem.

10. (c). For local minima we first put $f'(x) = 0$, then we will get $x = -1$. Since $f''(x) = 2 \geq 0$, that implies $x = -1$ is local minimizer. Since it is a convex function it is also a global minimizer.