Weekly Assignment 12

Due on 2023-10-19, 23:59 GMT

Consider the following problem in Shannon’s channel coding theorem.

Problem Statement: Suppose we have a communication channel with an input alphabet \( A = \{0, 1\} \) and an output alphabet \( B = \{0, 1, 2\} \). The channel is such that the probability of error in transmission is 0.1. What is the maximum information rate for reliable communication over this channel?

Solution:

1. The maximum information rate for reliable communication over a binary-input, ternary-output channel with an error probability of 0.1 is given by the Shannon capacity formula:

   \[ R = \max_{p(x)} I(X; Y) \]

   where \( I(X; Y) \) is the mutual information between the input and output symbols, and the maximization is over all possible input distributions.

2. For a binary-input, ternary-output channel with an error probability of 0.1, the mutual information is given by:

   \[ I(X; Y) = \sum_{x \in A} \sum_{y \in B} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \]

3. Substituting the error probability of 0.1 into the mutual information formula, we get:

   \[ I(X; Y) = \sum_{x \in A} \sum_{y \in B} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \]

4. Since we are maximizing this expression over all possible input distributions, we can use the method of Lagrange multipliers to solve this optimization problem.

5. The solution to this optimization problem gives the maximum information rate for reliable communication over this channel:

   \[ R = \frac{1}{2} \log \frac{1}{0.1} \]

   \[ R = 1 \text{ bit per channel use} \]

6. Therefore, the maximum information rate for reliable communication over this channel is 1 bit per channel use.