Assignment 09

The due date for submitting this assignment has passed. For your own records you have not submitted this assignment.

Instructions:
1. Submit all questions.
2. Include page number and question number.
4. You have 3 hours between 7am and 10am to submit this assignment.

1. Let \( f \) be an integral operator of \( (\mathcal{D}, \mathcal{H}) \). Is it possible to express \( f \) as \( f(x) = \int \mathcal{D} f(x, y) g(y) \, dy \)?

   - True
   - False
   - 2 points

2. Let \( \mathcal{H} \) be a Reproducing Kernel Hilbert Space (RKHS) equipped with the following kernel function:
   \[
   K(x, y) = \begin{cases} 
   1, & x = y \\
   0, & x \neq y
   \end{cases}
   \]

   a. Is \( \mathcal{H} \) a RKHS?
      - True
      - False
      - 2 points

   b. Is \( \mathcal{H} \) a Hilbert space?
      - True
      - False
      - 2 points

3. Let \( K \in \mathcal{R} \) be a positive definite kernel function. Is \( K \) a symmetric function?
   - True
   - False
   - 2 points

4. Let \( \mathcal{X} \) be a measurable space and \( \mathcal{S} \) be a measurable space. Is \( \mathcal{X} \times \mathcal{S} \) a measurable space?
   - True
   - False
   - 2 points

5. Let \( \mathcal{H} \) be a Reproducing Kernel Hilbert Space (RKHS). Is \( \mathcal{H} \) a Banach space?
   - True
   - False
   - 2 points

6. Let \( \mathcal{H} \) be a Reproducing Kernel Hilbert Space (RKHS) equipped with the following kernel function:
   \[
   K(x, y) = \begin{cases} 
   1, & x = y \\
   0, & x \neq y
   \end{cases}
   \]

   a. Is \( \mathcal{H} \) a RKHS?
      - True
      - False
      - 2 points

   b. Is \( \mathcal{H} \) a Hilbert space?
      - True
      - False
      - 2 points

7. Let \( \mathcal{X} \) be a measurable space and \( \mathcal{Y} \) be another measurable space. Is \( \mathcal{X} \times \mathcal{Y} \) a measurable space?
   - True
   - False
   - 2 points

8. Let \( \mathcal{H} \) be a Reproducing Kernel Hilbert Space (RKHS). Is \( \mathcal{H} \) a Banach space?
   - True
   - False
   - 2 points