Assignment 08

The due date for submitting this assignment has passed. Due on 2019-03-27, 23:59 IST.

As per our records you have not submitted this assignment.

Instructions:
1. Attempt all questions.
3. Solutions to be posted: 28th March 2019
4. Older browsers might show unnecessary vertical bars at the end of math equations

1) Consider the polyphase representation of a 16-channel filter bank. Suppose the product $E(z)R(z)$ is an identity matrix, we get the reconstructed output after a delay of $x$ units. The value of $x$ is

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 15

2) Consider a two channel filter bank with quadrature mirror property satisfying alias cancellation conditions. Let $H_0(z)$ and $H_1(z)$ be the analysis filters and $F_0(z)$ and $F_1(z)$ be the synthesis filters.

Let $F_0(z) = H_0(z)$. For which of the following matrices is the vector $[H_0(z) H_1(z) F_0(z) F_1(z)]^T$ an eigenvector with eigenvalue 1?
No, the answer is incorrect.
Score: 0

Accepted Answers:

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
\end{bmatrix}
\]

3) Consider the following $M$-channel delay filter bank. Let $H_i(z)$ and $F_i(z)$ denote the analysis and the synthesis filter in the $(i+1)^{th}$ branch (where $i \in \{0,1,\ldots,M-1\}$) respectively.

Which of the following are the expressions for $H_i(z)$ and $F_i(z)$?

- $H_i(z) = z^{-i}$ and $F_i(z) = z^{-i}$
- $H_i(z) = z^{-(i-1)}$ and $F_i(z) = z^{-(i-1)}$
- $H_i(z) = z^{-i}$ and $F_i(z) = z^{(i-M+1)}$
- $H_i(z) = z^{-(i-1)}$ and $F_i(z) = z^{(i-M)}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$H_i(z) = z^{-i}$ and $F_i(z) = z^{(i-M+1)}$

4) Which of the following are half band filters?

- $H(z) = 7 + 4z^{-1} - z^{-5}$
5) Which of the following filters are minimum phase filters? 2 points

- \( H_1(z) = \frac{3 - z^{-1}}{2 - 5z^{-1} + 2z^{-2}} \)
- \( H_2(z) = \frac{6 - 7z^{-1} + 2z^{-2}}{3 + 2z^{-1}} \)
- \( H_3(z) = \frac{1 - 2z^{-1}}{5 - 3z^{-1}} \)
- \( H_4(z) = \frac{3 + 7z^{-1} + 2z^{-2}}{1 - 2z^{-1}} \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
- \( H_1(z) = \frac{3 - z^{-1}}{2 - 5z^{-1} + 2z^{-2}} \)
- \( H_2(z) = \frac{6 - 7z^{-1} + 2z^{-2}}{3 + 2z^{-1}} \)

6) (True/False): As \( P(z) = I \) ensures perfect reconstruction, for \( E(z) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ -3z^{-1} & 2 & 1 \end{bmatrix} \), one can obtain stable synthesis filters using \( R(z) = E(z)^{-1} \) which yield perfect reconstruction.

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

7) For the filter bank in Question 3, what is \( A_i(z) \)? 2 points

- \( A_1(z) = 0 \)
- \( A_i(z) = z^{-(M-1)} \delta_i \)
- \( A_i(z) = z^{-M} \)
- \( A_i(z) = z^{-(M-1)} \frac{M}{M} \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
- \( A_i(z) = z^{-(M-1)} \delta_i \)

8) (True/False) In Question 3, \( x[n] \) can be reconstructed perfectly from the filter bank output \( x[n] \) by using delay elements. 2 points
9) Consider the Haar wavelet decomposition and reconstruction up to second scale as non-uniform filter bank (i.e., decimation and upsampling rates are non-uniform across different channels). The analysis filters look like the following on simplification:

The analysis filters \( H_i(z) \) for this filter bank are given as follows:

\[
H_1(z) = \frac{1 + z^{-1} + z^{-2} + z^{-3}}{2},
\]

\[
H_2(z) = \frac{1 + z^{-1} - z^{-2} - z^{-3}}{2},
\]

\[
H_0(z) = \frac{1 - z^{-1}}{\sqrt{2}}.
\]

Test which of the special properties given below are satisfied by the analysis filter bank.

- Strictly complementary
- Power complementary
- All-pass complementary
- None of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
None of the above

10) Consider a 3 channel filter bank with analysis filters \( H_0(z) = 1, H_1(z) = 6 + z^{-1} + 6z^{-3} \) and \( H_2(z) = 2 + z^{-1} + 2z^{-2} \) and synthesis filters \( F_0(z) = -1 - z^{-1} + 2z^{-2} + 4z^{-3} - 5z^{-4}, F_1(z) = -1 + z^{-1} \) and \( F_2(z) = 1 - z^{-4} \). Which of these following choices is \( E(z) \), \( R(z) \) and do these form Nyquist M filters?

\[
E(z) = \begin{bmatrix} 1 & 6 & 2 \\ 0 & 1 & 1 \\ 0 & 6z^{-3} & 2 \end{bmatrix}, \quad R(z) = \begin{bmatrix} 2 - 5z^{-3} & -1 + 4z^{-3} & -1 \\ 0 & 1 & -1 \\ 0 & -z^{-3} & 1 \end{bmatrix}
\]

and they are Nyquist M filter

\[
E(z) = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 6z^{-1} \\ 2 & 1 & 2 \end{bmatrix}, \quad R(z) = \begin{bmatrix} 2 - 5z^{-1} & 0 & 0 \\ -1 + 4z^{-1} & 1 & -z^{-1} \\ -1 & -1 & 1 \end{bmatrix}
\]

and they are not Nyquist M filter

\[
E(z) = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 6z^{-1} \\ 2 & 1 & 2 \end{bmatrix}, \quad R(z) = \begin{bmatrix} -1 & -1 + 4z^{-1} & 2 - 5z^{-1} \\ -1 & 1 & 0 \\ 1 & -z^{-1} & 0 \end{bmatrix}
\]

and they are not Nyquist M filter

\[
E(z) = \begin{bmatrix} 1 & 6 & 2 \\ 0 & 1 & 1 \\ 0 & 6z^{-1} & 2 \end{bmatrix}, \quad R(z) = \begin{bmatrix} -1 & -1 + 4z^{-1} & 2 - 5z^{-1} \\ -1 & 1 & 0 \\ 1 & -z^{-1} & 0 \end{bmatrix}
\]

and they are Nyquist M filter

No, the answer is incorrect.
Score: 0
Accepted Answers:

\[
E(z) = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 6z^{-1} \\ 2 & 1 & 2 \end{bmatrix}, \quad R(z) = \begin{bmatrix} 2 - 5z^{-1} & 0 & 0 \\ -1 + 4z^{-1} & 1 & -z^{-1} \\ -1 & -1 & 1 \end{bmatrix}
\]

and they are not Nyquist M filter