Introduction to Time-Varying Electrical Networks: Week 7

Problem 1

Fig. 1 shows an RC network with a periodically time-varying resistor. The capacitor \( C = 1 \, \text{F} \). The conductance varies from 0 to \( 2\pi 1000 \, \text{S} \) as shown in the figure. Write a MATLAB program to determine the harmonic transfer functions of this network. Assume that \( g(t) \) has a rise- and fall-times of 0.1 s and an average value that is half its peak value. Denoting the maximum number of harmonics at any node by \( K \), so that there are \((2K + 1)\) sinusoids at any node, your code should be able to accommodate a user-specified \( K \). For uniformity, the range of the y-axis must be 0-1, and the x-axis from -3 to 3 Hz, with increments in \( f \) chosen to be 0.05. What do you notice as you change \( K \)? Run sanity checks for the values of the harmonic transfer functions at frequencies 1, 2, 3 Hz.

Suggestion: When you invert the \( G \) matrix in MATLAB to solve \( GV = I \), do not use \( V = \text{inv}(G) \times I \). Turns out that matrix inversion is a very computationally intensive process – remember that you are inverting a \( 3000 \times 3000 \) matrix. Rather, use \( V = G \backslash I \), which is much quicker. Since I told you not to do something, I am sure you will definitely do it. See for yourself how much quicker the \( V = G \backslash I \) is with respect to explicitly computing the inverse.

Problem 5

Next, use the code you developed in the previous problem to compute \( H_{-k}(j2\pi k) \) for 1, 2. Use \( K = 512 \), and a frequency resolution of 0.01 Hz. Plot \( H_{-k}(j2\pi f) \) for 1, 2 for \( f \) between 0 and 3 Hz at intervals of every 0.01 Hz. What do you notice? For ease of computation, assume that the rise- and fall-times of \( s(t) \) are 1% of its period, and that the average of \( s(t) \) is 25% of its peak value.

Fig. 2 shows an RC network with a periodically-operated ideal switch. The switch is closed when \( s(t) \) is high and open when it is low. \( s(t) \) has a frequency of 1 Hz and a 25% duty cycle. The capacitor \( C = 1 \, \text{F} \). \( RC = 10 \, \text{s} \). The output is the voltage at the output of the capacitor \( v_c(t) \). Analytically determine \( H_{-k}(j2\pi k) \) for 1, 2 (use the fact that \( RC \gg T_s/4 \) to advantage).