Assignment 7

The due date for submitting this assignment has passed. Due on 2021-03-10, 23:59 IST.

As per our records you have not submitted this assignment.

Inner Product, Norm, Orthonormal Basis, Gram-Schmidt Orthonormalisation, Linear Functionals, Orthogonal Complements

1) Which of the following are valid inner products on the given vector spaces?  

- \[ \langle u, v \rangle' = \langle u + v, u - v \rangle, \text{ where } \langle \cdot, \cdot \rangle \text{ is a valid inner product.} \] 1 point
- \[ \langle p, q \rangle = \int_0^2 p(x)q(x)dx \text{ on } \mathbb{P}_2(\mathbb{R}), \text{ the vector space of all polynomials with degree } \leq 2 \]  
- \[ \langle (a, b), (c, d) \rangle = ac - bd \text{ on } \mathbb{R}^2 \]  
- \[ \langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 - 2x_1y_2 - 2x_2y_1 + 6x_2y_2 \text{ on } \mathbb{R}^2 \]  

No, the answer is incorrect. 
Score: 0 

Accepted Answers:
- \[ \langle p, q \rangle = \int_0^2 p(x)q(x)dx \text{ on } \mathbb{P}_2(\mathbb{R}), \text{ the vector space of all polynomials with degree } \leq 2 \]  
- \[ \langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 - 2x_1y_2 - 2x_2y_1 + 6x_2y_2 \text{ on } \mathbb{R}^2 \]

2) Let \( v_1, v_2 \) be two vectors in a vector space \( V \) with norm \( \| v_1 \| = \| v_2 \| = 3 \) and inner product 

\[ \langle v_1, v_2 \rangle = 3 + 4j. \]  

What is the value of \( \langle v_1 + 3v_2, v_1 + (-2j)v_2 \rangle \)?
3) Let \( B = \{u_1, u_2\} \) be a basis for a two-dimensional inner-product space \( V \), where \( u_1 \) and \( u_2 \) are unit-norm vectors and \( \langle u_1, u_2 \rangle = \frac{1}{3} \).

The coordinates of the vectors \( v_1, v_2 \in V \) under the basis \( B \) are \((1, k)\) and \((5, 1)\), respectively. What is the value of \( k \) that makes \( v_1 \) and \( v_2 \) orthogonal to each other?

No, the answer is incorrect. Score: 0
Accepted Answers: 10 + 48j

4) Let \( p(\cdot) \in \mathbb{P}_2(\mathbb{R}) \) be a polynomial such that
\[
\int_{0}^{1} p^2(x) \, dx = 9.
\]

What is the maximum possible value of \( \int_{-1}^{1} p(x)(1 - 3x^2) \, dx \) ? (\( \mathbb{P}_2(\mathbb{R}) \) is the vector space of all polynomials with degree \( \leq 2 \).)

No, the answer is incorrect. Score: 0
Accepted Answers: 
- 9
- \( \frac{6}{\sqrt{5}} \)
- \( \frac{36}{5} \)
- 3

5) Let a polynomial \( q(\cdot) \in \mathbb{P}_2(\mathbb{R}) \) be such that \( p(1) = \int_{-1}^{1} p(x)q(x) \, dx \) for every \( p \in \mathbb{P}_2(\mathbb{R}) \). If \( q(x) = ax^2 + bx + c \), What is the value of \( 2(a - b - c) \) ? (\( \mathbb{P}_2(\mathbb{R}) \) is the vector space of all polynomials with degree \( \leq 2 \)).
6) Let $A$ be a real matrix of size $4 \times 6$ and have its maximum possible rank. What is the dimension of $(\text{null } A)^\perp$? ($U^\perp$ refers to the orthogonal complement of a set $U$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 6

1 point

7) Consider the subspace $U = \text{span}\{(1, 2, 3, 8), (1, 3, 4, 11)\}$ in $\mathbb{R}^4$. Which of the following is an orthonormal basis for $U^\perp$? Assume the inner product to be the usual dot product. ($U^\perp$ refers to the orthogonal complement of a set $U$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 4

1 point

8) Which of the following form an orthonormal basis of $\mathbb{R}^3$? (Assume usual dot product)

No, the answer is incorrect.
Score: 0
Accepted Answers:

1 point
9) Which of the following are NOT valid norms in $\mathbb{R}^3$ over $\mathbb{R}$? 

\[
\|x\| = |x_1 + x_2 + x_3| \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

\[
\|x\| = |x_1| + |x_2| + |x_3| \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

\[
\|x\| = |x_1^3 + x_2^2 + x_3^2| \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

\[
\|x\| = x_1^3 + x_2^3 + x_3^3 \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[
\|x\| = |x_1 + x_2 + x_3| \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

\[
\|x\| = |x_1^3 + x_2^3 + x_3^3| \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

\[
\|x\| = x_1^3 + x_2^3 + x_3^3 \quad \forall x = (x_1, x_2, x_3) \in \mathbb{R}^3
\]

10) Let $U$ be a subset of some finite dimensional vector space $V$. Which of the following statements are NOT necessarily valid? ($U^\perp$ refers to the orthogonal complement of a set $U$)

\[ U \text{ is a subspace if and only if } U \cap U^\perp = \{0\}. \]

\[ U^\perp \text{ is a subspace only if } U \text{ is a subspace.} \]

\[ U + U^\perp = V \text{ if } U \text{ is any subset of vector space } V. \]

\[ \text{if } U \text{ and } W \text{ are subspaces of } V, \text{ then } U \cap W = (U^\perp + W^\perp)^\perp \]

No, the answer is incorrect.

Score: 0

Accepted Answers:

\[ U \text{ is a subspace if and only if } U \cap U^\perp = \{0\}. \]

\[ U^\perp \text{ is a subspace only if } U \text{ is a subspace.} \]

\[ U + U^\perp = V \text{ if } U \text{ is any subset of vector space } V. \]

11) Let $a, b, c, d$ be positive real numbers. Find the minimum possible value of

\[
(a + b + c + d) \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)
\]

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 16

1 point

12) Consider the functions $h_1(x) = x$ and $h_2(x) = e^x$ in the vector space $C[0, 1]$ (the set of all continuous functions $f : [0, 1] \rightarrow \mathbb{R}$) with the inner product defined as,

\[ \langle f(x), g(x) \rangle = \int_0^1 f(x)g(x)dx. \]

Calculate $\langle h_1, h_2 \rangle$. 

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https://onlinecourses.nptel.ac.in/noc21_ee38/unit/unit=39&assessment=106
No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 1