Assignment 6

The due date for submitting this assignment has passed. **Due on 2021-03-03, 23:59 IST.**

As per our records you have not submitted this assignment.

Properties of Eigenvalues, Linear State Space Equations, Discrete time Linear Systems & DFT, Sequences & counting paths, PageRank Algorithm

1) Consider the matrix

\[ A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}, \]

where \( a, b \in \mathbb{R} \). What are the eigenvalues of the matrix \( A \)? Are such matrices always diagonalizable? (for any value of \( a \) and \( b \))

- \((a + b, a + b)\); No, \( A \) need not be diagonalizable.
- \((a + b, a - b)\); Yes, \( A \) is always diagonalizable.
- \((a + b, a + b)\); Yes, \( A \) is always diagonalizable.
- \((a - b, a - b)\); No, \( A \) need not be diagonalizable.

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \((a + b, a - b)\); Yes, \( A \) is always diagonalizable.

2) Let a linear map \( T \) be represented by the matrix

\[ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \]

where \( a, b \in \mathbb{R} \). What is the eigenvalue of \( T \)?

- \(1\); Yes, \( T \) is always diagonalizable.
- \(2\); No, \( T \) need not be diagonalizable.

No, the answer is incorrect.

Score: 0

Accepted Answers:
- \(1\); Yes, \( T \) is always diagonalizable.
Discrete Fourier Transforms (unit? unit=38&lesson=70)

Sequences and counting paths in graphs (unit? unit=38&lesson=71)

PageRank Algorithm (unit? unit=38&lesson=72)

Applied Linear Algebra:
Week 6 Feedback Form (unit? unit=38&lesson=49)

Quiz : Week 6 Tutorial (Non Graded) (assessment? name=108)

Quiz : Assignment 6 (assessment? name=104)

Assignment 6 solution (unit? unit=38&lesson=110)

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$$M = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

with respect to some basis. Which of the following statements are correct?

- [ ] Sum of eigenvalues of $T$ is 12.
- [ ] Sum of eigenvalues of $T$ is 9.
- [ ] $T$ is not invertible.
- [ ] Null space of $T$ is $\{0\}$.
- [ ] $T^{-1}$ exists, and 0.2 is an eigenvalue of $T^{-1}$.
- [ ] $T^{-1}$ exists, and product of eigenvalues of $T^{-1}$ is $\frac{1}{15}$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
- Sum of eigenvalues of $T$ is 9.
- Null space of $T$ is $\{0\}$.
- $T^{-1}$ exists, and 0.2 is an eigenvalue of $T^{-1}$.
- $T^{-1}$ exists, and product of eigenvalues of $T^{-1}$ is $\frac{1}{15}$.

3) Which of the following matrices represent a stable system?  

1 point

$$\begin{bmatrix} 0.7 & 0 & 0 \\ 3 & 0.4 & 0 \\ 8 & 9 & -0.3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & -0.5 & 0 \\ 0 & 7 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.1 & 0 & 0 \\ 4 & 0.4 & 0 \\ 0 & -6 & -1.3 \end{bmatrix}$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
- $\begin{bmatrix} 0.7 & 0 & 0 \\ 3 & 0.4 & 0 \\ 8 & 9 & -0.3 \end{bmatrix}$
Let \( \{a_k\} \) be a sequence such that \( a_k = \frac{7}{4}a_{k-1} - \frac{7}{8}a_{k-2} + \frac{1}{8}a_{k-3}, \ k \geq 4 \). The following values are known: \( a_3 = 4, a_2 = 2, a_1 = 1 \). Which of the following is the general expression for the \( n \)-th element \( a_n \) of the sequence? (You may use a software tool like MATLAB for calculations)

- \( a_n = 3.000628 - 30.997678 \times 2^{-n} + 8.988141 \times 4^{-n} \forall n \geq 4 \)
- \( a_n = 7.000628 - 27.997678 \times 2^{-n} + 31.988141 \times 4^{-n} \forall n \geq 4 \)
- \( a_n = 6.000628 - 20.997678 \times 2^{-n} + 31.988141 \times 4^{-n} \forall n \geq 4 \)
- \( a_n = 7.000628 - 17.997678 \times 2^{-n} + 5.988141 \times 4^{-n} \forall n \geq 4 \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(1 point)

5) Let 2, 5 and 9 be the eigenvalues of a matrix \( M \). Find the eigenvalue of the matrix \( A = (M^3 - M^2 + 6M) \) with the highest magnitude.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 702

6) Let 0, 3, and 5 be the eigenvalues of a 3x3 matrix \( M \) representing a linear operator \( T \) with respect to some basis. What is the sum of nullity and left-nullity of \( M \)?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 2

7) Consider the matrices

\[
C_a = \begin{bmatrix}
-1 & 2 & 1 & 2 \\
2 & -1 & 2 & 1 \\
1 & 2 & -1 & 2 \\
2 & 1 & 2 & -1
\end{bmatrix},
C_b = \begin{bmatrix}
3 & -1 & 3 & -5 \\
-5 & 3 & -1 & 3 \\
3 & -5 & 3 & -1 \\
-1 & 3 & -5 & 3
\end{bmatrix}.
\]

What are the eigenvalues of the matrix \( C_a C_b \)?

- 0, \(-8j\), 48, \(8j\)
- 0, \(-8j\), \(-48\), \(8j\)
- 0, \(8j\), 48, \(-8j\)
-48, -8j, 48, 8j

No, the answer is incorrect.  
Score: 0  
Accepted Answers:  
0, -8j, -48, 8j

8) Let \( \hat{x} = [\hat{x}(0) \ \hat{x}(1) \ \hat{x}(2) \ \hat{x}(3)]^T \) be the DFT of the vector \( x = [1 \ 2 \ 3 \ 4]^T \). What is the value of \( \hat{x}(2) \)?

No, the answer is incorrect.  
Score: 0  
Accepted Answers:  
(Type: Numeric) -1

9) Find the eigenvalues of the matrix
\[
A = \begin{bmatrix}
112 & -34 & -26 \\
-168 & 51 & 39 \\
55 & -18 & -14 \\
\end{bmatrix}.
\]

Try to find them using properties without a direct computation! What is the largest eigenvalue of \( A \)?

No, the answer is incorrect.  
Score: 0  
Accepted Answers:  
(Type: Numeric) 150

10) Suppose a matrix \( A \) has eigenvalues \(-\frac{1}{2}, \frac{1}{4} \) and 1. Let the eigenvalues of the matrix \( B = (A^{-1} - 3I)^T + 5I \) be \( p, q, r \) with \(|p| \leq |q| \leq |r| \). What is the value of \( \frac{2p+r}{q} \)? Is the matrix \( B \) invertible?

- 1. \( B \) is invertible.  
- 2. \( B \) is non-invertible.

No, the answer is incorrect.  
Score: 0  
Accepted Answers:  
2. \( B \) is non-invertible.

11) Let a linear state-space system be defined as follows:
\[
x_{k+1} = Ax_k,
\]
where
\[
A = \begin{bmatrix}
0.2 & 0.8 \\
0.8 & 0.2 \\
\end{bmatrix}
\]
and let \( x_0 = (1, 0)^T \). What is the steady state value of \( x_k \) as \( k \to \infty \)?
12) Suppose \( v \) and \( u \) are non-zero column vectors. Let the matrix \( A \) be defined as 
\[
A = uv^T.
\]
Which of the following is (are) true?

- \( v \) is an eigenvector of \( A \)
- \( u \) is an eigenvector of \( A \)
- \( A \) has rank 1
- None of these

No, the answer is incorrect.
Score: 0
Accepted Answers:
\( u \) is an eigenvector of \( A \)
\( A \) has rank 1