Assignment 2

The due date for submitting this assignment has passed.

Due on 2021-02-07, 23:59 IST.

As per our records you have not submitted this assignment.

Linear Maps & Matrices, Null space, Range & Fundamental theorem, Algebraic operations, Column space, Rank

1) Let \( T : \mathbb{R}^4 \rightarrow \mathbb{R}^2 \) be a linear map. Given that \( T(0, 0, 0, 1) = (1, 0); T(0, 0, 1, 0) = (1, 1); T(1, 1, 0, 0) = (0, 0); T(0, 1, 1, 0) = (3, 4), \) find \( T(1, 7, 3, −2). \)

- (10, 20)
- (13, 21)
- (15, 17)
- (17, 10)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(13, 21)

2) Matrix \( M \) (given below) represents a linear map \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) with respect to some bases.

\[
\begin{pmatrix}
10 & 11 & 12 \\
13 & 14 & 15 \\
16 & 17 & 18
\end{pmatrix}
\]
The linear map $T$ is __________.

- Injective
- Surjective
- Injective & Surjective
- Neither Injective nor Surjective

No, the answer is incorrect.
Score: 0
Accepted Answers:  
Neither Injective nor Surjective

3) Which of the following form valid linear maps? There may be more than one correct 1 point answer. Assume $T : \mathbb{R}^2 \to \mathbb{R}^2$.

- $T(x, y) = (2x, 3y)$
- $T(x, y) = (y, x)$
- $T(x, y) = \left(\frac{x^2+1}{x}, \frac{y^2+1}{y}\right)$
- $T(x, y) = (x^2, y^3)$

No, the answer is incorrect.
Score: 0
Accepted Answers:  
$T(x, y) = (2x, 3y)$  
$T(x, y) = (y, x)$

4) Let $A$ be an $m \times n$ matrix with rank $r$. Which of the following matrices also have 1 point rank $r$? Option (a) is a $2m \times n$ matrix with two $A$ matrices stacked on top of each other. Option (b) is a $m \times 2n$ matrix with two $A$ matrices beside each other.

- $egin{bmatrix} A \\ A \end{bmatrix}$
- $egin{bmatrix} A & A \\ A & 0 \end{bmatrix}$
- All of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:  
$egin{bmatrix} A \\ A \end{bmatrix}$  
$egin{bmatrix} A & A \end{bmatrix}$

5)
Find 'a' such that matrix $M$ is NOT injective. Type the value of $a$ in the box below.

$$M = \begin{bmatrix} a & 3 & 4 \\ 2 & 7 & -9 \\ 1 & -6 & 5 \end{bmatrix}$$

No, the answer is incorrect.
Score: 0
Accepted Answers: (Type: Range) -7.1, -6.9

6) Let a linear map $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be represented by the matrix $M$ given below with respect to standard basis.

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 4 \end{bmatrix}.$$  
Which of the following are sub-spaces of $\mathbb{R}^3$ that have dimension 2?

- range($T$)
- null($T$)
- span {first two rows of $M$}
- span {last two columns of $M$}

No, the answer is incorrect.
Score: 0
Accepted Answers: span {first two rows of $M$}, span {last two columns of $M$}

7) Which of the following are valid injective linear maps? Select all correct options.

- $T : P_n(\mathbb{R}) \rightarrow P_{n+2}(\mathbb{R})$ over field $\mathbb{R}$, $T(p(x)) = x^2p(x)$
- $T : \mathbb{R}^n \rightarrow \mathbb{R}^{n-1}$, $T((x_1, x_2, \ldots, x_n)) = (x_2, x_3, \ldots, x_n)$
- $T : \mathbb{R}^n \rightarrow \mathbb{R}^{n+1}$, $T((x_1, x_2, \ldots, x_n)) = (0, x_1, x_2, x_3, \ldots, x_n)$
- $T : P_n(\mathbb{R}) \rightarrow P_n(\mathbb{R})$, $T(p(x)) = (p(x) + 2)$

No, the answer is incorrect.
Score: 0
Accepted Answers: $T : P_n(\mathbb{R}) \rightarrow P_{n+2}(\mathbb{R})$ over field $\mathbb{R}$, $T(p(x)) = x^2p(x)$, $T : \mathbb{R}^n \rightarrow \mathbb{R}^{n+1}$, $T((x_1, x_2, \ldots, x_n)) = (0, x_1, x_2, x_3, \ldots, x_n)$

8) Select the correct options:

- Let $M(T)$ be an $m \times n$ matrix representing a linear map from $F^n \rightarrow F^m$. If $M$ is a wide matrix, that is it has more columns than rows ($m < n$), then $T$ can be injective.
Let $M(T)$ be an $m \times n$ matrix representing a linear map from $F^n \rightarrow F^m$. If $M$ is a tall matrix, that is it has more rows than columns ($m > n$), then $T$ can be injective.

It is possible for a linear map to be injective and have a non-trivial null space.

Consider finite dimensional vector spaces $V$, $W$, and linear map $T : V \rightarrow W$. Let $T$ be represented by matrix $M$. If the column space of $M$ is equal to $W$, then $T$ is surjective.

No, the answer is incorrect.
Score: 0
Accepted Answers:

Let $M(T)$ be an $m \times n$ matrix representing a linear map from $F^n \rightarrow F^m$. If $M$ is a tall matrix, that is it has more rows than columns ($m > n$), then $T$ can be injective.

Consider finite dimensional vector spaces $V$, $W$, and linear map $T : V \rightarrow W$. Let $T$ be represented by matrix $M$. If the column space of $M$ is equal to $W$, then $T$ is surjective.

9) Let $T : P_1(\mathbb{R}) \rightarrow P_1(\mathbb{R})$, where $P_1(\mathbb{R})$ is the set of all polynomials with degree $\leq 1$ and real coefficients, be a linear map. Let $T(x + 2) = (3x + 2)$ and $T(3x) = (-x + 6)$. If $T(3x + 6) = (ax + b)$, what is the value of $a + b$?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 14.99, 15.01

10) Consider the matrix multiplication $C = AB$ where:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 90 & 60 & 30 \\ 80 & 50 & 20 \\ 70 & 40 & 10 \end{bmatrix}$$

Which of the following when evaluated give the same answer $C$? There may be more than one correct answer.

No, the answer is incorrect.
Score: 0
Consider the matrix
\[
A = \begin{bmatrix}
1 & 5 & 7 \\
2 & 3 & 0 \\
6 & -1 & -20
\end{bmatrix}.
\]
Let \{(1, 1, a), (0, 1, b)\} be a basis for the column space of \(A\). What is the value of \(a + 2b\)? Enter your answer rounded to two decimal places.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 10.1, 10.9

1 point

12) Find the dimension of the null space of linear map \(T\) represented by the following matrix
\[
M: \quad M = \begin{bmatrix}
1 & 6 \\
5 & 3 \\
-2 & 8
\end{bmatrix}
\]

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.01, 0.01

1 point