Assignment 11

The due date for submitting this assignment has passed. Due on 2021-04-07, 23:59 IST.
As per our records you have not submitted this assignment.

Positive operators, Quadratic Forms, Matrix Norms, Optimization, Isometries

1) Let $S$ be a $2 \times 2$ matrix such that $x^T S x = 3x_1^2 + 2x_1x_2 + 3x_2^2$ for all $x \in \mathbb{R}^2$. If the maximum and minimum of $\frac{x^T S x}{x^T x}$ are $a$ and $b$ respectively, find the value of $a + 2b$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 8

2) Let $R$ be a linear operator. Which of the following is(are) true?

- $T = RR^*$ must be a positive operator
- $T = RR^* + I$ must be an invertible operator.
- $T = \frac{R + R^*}{2}$ must be a positive operator if $R$ is a positive operator.
- $T = RR^* - I$ must be a positive operator

No, the answer is incorrect.
Score: 0
Accepted Answers:
3) Consider the following statements about linear operators over a complex inner product space. Which of them are true?

- All the eigenvalues of an isometry must be of magnitude 1.
- Every normal operator is an isometry.
- Every isometry is a normal operator.
- All isometries are invertible

No, the answer is incorrect.
Score: 0
Accepted Answers:
- All the eigenvalues of an isometry must be of magnitude 1.
- Every isometry is a normal operator.
- All isometries are invertible

4) Let $Q$ be a normal operator over a complex inner product space $V$ with eigenvalues $1, e^{\frac{i\pi}{3}}$ and $e^{-\frac{i\pi}{3}}$. If $v, u \in V$ with $\|v\| = \|u\| = 2$ and $\langle v, u \rangle = 3$, what is the value of $\langle Q(v + 2u), Q(v - u) \rangle$?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) -1

5) Consider the matrix

$$A = v_1v_1^T + v_2v_2^T$$

with

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \\ 1 \end{bmatrix} \quad \text{and} \quad v_2 = \begin{bmatrix} 3 \\ -3 \\ -\sqrt{2} \\ 1 \\ -\sqrt{2} \end{bmatrix}$$

Which of the following statements are true?

- $v^TAv \geq 0$ for any $v \in V$.
- $A$ has 1 as an eigenvalue.
- $A$ represents an isometry.
- $A^2 = A$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
- $v^TAv \geq 0$ for any $v \in V$. 

https://onlinecourses.nptel.ac.in/noc21_ee38/unit?unit=43&assessment=117
A has 1 as an eigenvalue.

\[A^2 = A.\]

6) Let \( A \) be a symmetric matrix which is also an isometry. Which of the following could 1 point be an eigenvalue for \( A \)?

- [ ] 1
- [ ] -1
- [ ] 2
- [ ] 0
- [ ] \( j \)

No, the answer is incorrect.
Score: 0
Accepted Answers:
1
-1

7) Find the matrix norm \( ||A|| \) of the following matrix: (write your answer correct to two decimal places)

\[ A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.6, 2.7

1 point

8) Which of the following operators are positive definite? (assume the field to be \( \mathbb{C} \))

Select the correct option(s):

- [ ] \[ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \]
- [ ] \[ \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \]
- [ ] \[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \]
- [ ] \[ \begin{bmatrix} 1 & i & 1+i \\ 1 & -i & 1-i \\ 1 & 2 & 1 \end{bmatrix} \]

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \]
9) Let $Q$ be the positive square root of the matrix

$$M = \begin{bmatrix}
50 & -14 & 0 \\
-14 & 50 & 0 \\
0 & 0 & 100
\end{bmatrix}.$$ 

Write the sum of the elements of $Q$ in the box below.

(For example the sum of the elements of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is $1 + 2 + 3 + 4 = 10$)

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 22

10) Let $V$ be a finite-dimensional inner product space over the real field $\mathbb{R}$.

$T : V \to V$ is an operator on $V$.

An operator $Q : V \to V$ is said to be a square root of $T$ if $Q^2 = T$.

Select the correct statement(s):

- There always exists a square root $Q$ for an operator $T$. (all operators $T$ have square roots)

- If an operator $T$ has a square root $Q$, then $Q$ is the unique square root of $T$

- $T$ can have a square root only if $T$ is positive semi-definite

- If $T$ has a positive semi-definite square root $Q$, then $T$ is self-adjoint

No, the answer is incorrect.
Score: 0
Accepted Answers:
if $T$ has a positive semi-definite square root $Q$, then $T$ is self-adjoint

11) Consider the following two operators over $\mathbb{R}^3$ represented in the standard basis:

$$A = \begin{bmatrix}
0.5 & 0.5 & 0 \\
0.5 & 0.5 & 0 \\
0 & 0 & -1
\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}
3.5 & 1.5 & 0 \\
1.5 & 3.5 & 0 \\
0 & 0 & 3
\end{bmatrix}$$

Select the correct option.

- $A \succ B$
- $A \prec B$
- $A = B$
- None of the above

No, the answer is incorrect.
Score: 0
12) Select the correct option(s):

- If $S$ is an isometry, so is $S^*$.  
- It is possible to construct a matrix such that the columns are orthonormal to each other, but the rows are not.
- If $S$ is an isometry, then $S = S^*$ ($S$ is self-adjoint).
- If $S$ is an isometry, then $S^* = S^{-1}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
- If $S$ is an isometry, so is $S^*$.
- If $S$ is an isometry, then $S^* = S^{-1}$.