Assignment 1

The due date for submitting this assignment has passed. Due on 2021-02-03, 23:59 IST.
As per our records you have not submitted this assignment.

Finite-dimensional vector spaces, Algebraic operations on vector spaces, Gaussian elimination

1) Which of the following is (are) valid subspace(s) of appropriate vector spaces? 1 point
Definitions of addition and scalar multiplication are as usual. (Note: \( P_k(\mathbb{R}) \) represents the vector space of all polynomials of degree \( \leq k \))

- The set of all polynomials \( p(x) \in P_4(\mathbb{R}) \) with no \( x^2 \) term

- The set of all \( 4 \times 4 \) skew-symmetric matrices (A with elements such that \( a_{ij} = -a_{ji} \forall i, j \)).

- The set of solutions \( (x, y, z) \) to the equation \( 5x + 3y + z = 1 \)

- The set of solutions \( y(t) \) to the differential equation \( y''(t) + 2y'(t) + 4y(t) = 0 \).

No, the answer is incorrect.
Score: 0
Accepted Answers:
The set of all polynomials \( p(x) \in P_4(\mathbb{R}) \) with no \( x^2 \) term
The set of all \( 4 \times 4 \) skew-symmetric matrices (A with elements such that \( a_{ij} = -a_{ji} \forall i, j \)).
The set of solutions \( y(t) \) to the differential equation \( y''(t) + 2y'(t) + 4y(t) = 0 \).

2)
Find a possible $k \in \mathbb{R}$ such that the following set $S$ is linearly dependent, given that the vectors $u, v$ and $w$ are linearly independent.

$$S = \text{span}\{(u + v + 2w), (3u + v - w), (-5u + kv + 4w)\}$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -1.01,-0.99

3) It is known that a vector space $V$ has a spanning set of size 4. The dimension of the vector space $V$ is necessarily:

- $4$
- $\leq 4$
- $\geq 4$
- $< 4$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\leq 4$

4) Let the coordinates of the vector $v$ in standard basis be equal to $(1, 3, 3)$. Consider the basis set $B = \{(1, 1, -1), (1, -1, 1), (-1, 1, 1)\}$ for the vector space $\mathbb{R}^3$. The coordinates of the vector $v$ under the basis set $B$ is given by:

- $(2, 2, 3)$
- $(1, 2, 2)$
- $(1, 1, 3)$
- B is not a basis set

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(2, 2, 3)$

5) Let $\{(1, 2), (2, a)\}$ be a spanning set for $\mathbb{R}^2$. Which of the following values can $a$ take? (There may be one or more valid choices for $a$).

-1
-4
4
2

No, the answer is incorrect.
Score: 0
Accepted Answers:
Let $M = \begin{bmatrix} -1 & 2 \\ -4 & 8 \\ 2 & 9 \end{bmatrix}$

Find $a + b + c$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.99, 2.01

7) Consider a finite-dimensional vector space $V$ defined over a scalar field $\mathbb{F}$. Determine which of the following statements are true, and choose the correct option.

i) The coordinates of any vector in a given basis is unique.

ii) $V$ has a unique basis set.

iii) A spanning set of $V$ with size $> \dim V$ will be linearly dependent.

iv) There are infinite different basis sets possible for a vector space.

- only (i) and (iii) are true
- only (i) and (iv) are true.
- (i), (iii) and (iv) are true.
- (i), (ii) and (iii) are true.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(i), (iii) and (iv) are true.

8) Consider two sets $A$ and $B$. $A$ is a basis for vector space $V$. $B$ is another basis for the same vector space $V$. Which of the following is true?

- $A$ and $B$ must have the same size.
- $A$ and $B$ need not have the same size. They could be the same or they could be different.
- $A$ and $B$ must have different sizes.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$A$ and $B$ must have the same size.

9) Let $U$ be a subspace of $\mathbb{R}^3$ defined as $\text{span}\{S\}$, where the set

$S = \{(1, 2, -2), (2, 3, 1), (-7, -12, 4), (1, 1, 3)\}$

Which of the following statement(s) about $U$ is (are) true?
10) Let $U$ and $V$ be subspaces of a vector space $W$. Which of the following are necessarily vector spaces? (There may be more than one correct option.)

- $U \cap V$
- $U \cup V$
- $U + V = \{u + v : u \in U, v \in V\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
- $U \cap V$
- $U + V = \{u + v : u \in U, v \in V\}$

11) Let $U$, $W$ be subspaces of $\mathbb{R}^3$, defined as $U = \text{span}\{(1, 2, 3), (1, 3, 2), (3, 8, 7)\}$, and $W = \text{span}\{(2, 4, 5), (1, 2, 1)\}$.
Let $P$ be the intersection $U \cap W$. The dimension of $P$ is equal to ____________.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 0.99,1.01

12) Consider the subspace of $V = \mathbb{R}^4$, $U = \text{span}\{(1, 2, 4, 1), (1, 0, 2, 3)\}$. Which of the following are sets $W$ such that $U \oplus W = V$? (There may be multiple correct answers)

- $W = \text{span}\{(2, 2, 6, 4), (1, 1, 1, 1)\}$
- $W = \text{span}\{(0, 0, 1, 0), (0, 0, 0, 1)\}$
- $W = \text{span}\{(1, 0, 3, 3), (0, 0, 0, 1)\}$
- $W = \text{span}\{(2, 0, 4, 6), (0, 0, 0, 1)\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
\[ W = \text{span}\{(0, 0, 1, 0), (0, 0, 0, 1)\} \]
\[ W = \text{span}\{(1, 0, 3, 3), (0, 0, 0, 1)\} \]