Assignment 3

The due date for submitting this assignment has passed.
At present no penalty has been assessed.

1. Which of the following vectors is a vector in $\mathbb{R}^4$?

- (A) $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$
- (B) $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$
- (C) $\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$
- (D) $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}$

2 points

2. If $A$ is a matrix such that $A^2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, which of the following statements is true?

- (A) $A$ is diagonalizable
- (B) $A$ is not diagonalizable
- (C) $A$ is a projection matrix
- (D) $A$ is an orthogonal matrix

2 points

3. The eigenvalues of a matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are $λ_1 = 1, λ_2 = 1, λ_3 = 0$. Which of the following is true?

- (A) $A$ is diagonalizable
- (B) $A$ is not diagonalizable
- (C) $A$ is a projection matrix
- (D) $A$ is an orthogonal matrix

2 points

4. If $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, what is the value of $P^2 BP$?

- (A) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$
- (B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
- (C) $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$
- (D) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

1 point

5. The trace and determinant of $A = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$ are $\text{trace}(A) = 4$ and $\text{det}(A) = 3$, respectively. Which of the following would be the eigenvalues of $A^T A$?

- (A) $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
- (B) $\begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$
- (C) $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$
- (D) $\begin{bmatrix} 4 & 1 \\ 0 & 1 \end{bmatrix}$

2 points

6. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, which of the following is true?

- (A) $A$ is diagonalizable
- (B) $A$ is not diagonalizable
- (C) $A$ is a projection matrix
- (D) $A$ is an orthogonal matrix

2 points

7. Which of the following statements are true?

- (A) The generalized eigenvalues of any square matrix are always independent
- (B) Similar matrices with diagonalized form will have orthogonal eigenvectors
- (C) The sum of the generalized multiplicities of all the eigenvalues of an square matrix is equal to the number of Jordan blocks of its Jordan form
- (D) The eigenvalues of a symmetric matrix are always distinct

2 points

8. What is the maximum of the singular values of $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?

- (A) $1$
- (B) $0.5$
- (C) $1.414$
- (D) $2$

1 point