Assignment 9

1. What is the system of a series form?

The system of a series form is defined as:

\[ S = C \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} x \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \]

where:
- \( S \): Output vector
- \( C \): Output matrix
- \( A \): State matrix
- \( B \): Input matrix
- \( x \): State vector
- \( D \): Direct transmission matrix

2. A system is given by the following transfer function:

\[ G(s) = \frac{1}{s^2 + 2s + 1} \]

a. Determine the poles and zeros of the system.

The poles of the system are found by setting the denominator to zero:
\[ s^2 + 2s + 1 = 0 \]

The characteristic equation is:
\[ s^2 + 2s + 1 = (s + 1)^2 \]

The pole is:
\[ s = -1 \]

There are no zeros because the numerator is constant:
\[ G(s) = \frac{1}{s + 1} \]

b. Determine the inverse Laplace transform of the system.

The inverse Laplace transform of the system is:
\[ g(t) = e^{-t} \]

3. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 1 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 = 1 \]

The poles are:
\[ \lambda = \pm i \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{1}{s^2 + 1} \]

4. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \]

\[ y = \begin{bmatrix} -1 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 - \lambda + 1 = 0 \]

The poles are:
\[ \lambda = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{-1}{s^2 - s + 1} \]

5. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 1 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 = 0 \]

The pole is:
\[ \lambda = 0 \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{1}{s} \]

6. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 = 0 \]

The poles are:
\[ \lambda = 0 \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{1}{s^2} \]

7. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 = 0 \]

The poles are:
\[ \lambda = 0 \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{1}{s^2} \]

8. A system is given by the following state-space model:

\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \]

a. Determine the poles of the system.

The characteristic equation is:
\[ \lambda^2 = 0 \]

The poles are:
\[ \lambda = 0 \]

b. Determine the transfer function of the system.

The transfer function is:
\[ G(s) = \frac{1}{s^2} \]