Assignment 12

1. Consider a system with the state-space given by:

\[
\begin{align*}
x' &= Ax + Bu \\
\end{align*}
\]

where \( A \) and \( B \) are given matrices. Determine the controllability of the system.

2. Consider the following differential equation:

\[
\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)
\]

Find the system's response to the input \( u(t) = \sin(\omega t) \) for \( \omega = 1 \) rad/s.

3. Consider the linear system given by:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
\end{align*}
\]

where \( A \) and \( B \) are given matrices. Show that the system is controllable if and only if the controllability matrix \( C = [B \ A B \ A^2 B \ \ldots \ A^{n-1} B] \) has full rank.

4. Consider the following dynamical system:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\
\end{align*}
\]

Find the system's response to the input \( u(t) = \cos(\omega t) \) for \( \omega = 1 \) rad/s.

5. Consider the linear system given by:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
\end{align*}
\]

where \( A \) and \( B \) are given matrices. Show that the system is observable if and only if the observability matrix \( O = [C \ B C \ AB C \ A^2 B C \ \ldots \ A^{n-1} B C] \) has full rank.